Geophys. J. Int. (1997) 131, 552–558

On the statistical mechanics of distributed seismicity

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Accepted 1997 June 26. Received 1997 June 16; in original form 1997 March 12

SUMMARY

In order to understand the underlying physics of distributed seismicity better we have considered a 2-D array of slider blocks connected by springs and interacting via static friction with a surface. There is no driving plate in this model. The time evolution of the system is found from numerical simulations in a cellular automata formulation. Energy is conserved and is the single control parameter. The distribution of energies in the springs is found to obey a modified Maxwell–Boltzmann statistics. It is found that the number–size statistics of clusters of unstable sliding blocks is identical to those in percolation clusters in the site-to-site percolation model. There is a well-defined critical point when unstable blocks become connected across the array. It has been previously suggested that distributed seismicity in a seismic zone is the percolation backbone of a 3-D percolation cluster. The fact that low-level seismicity satisfies the Gutenberg–Richter frequency–magnitude relation and is nearly constant in time also suggests that this background seismicity is similar to thermally induced noise.

Key words: earthquakes, geostatistics, seismic modelling, seismology, statistical methods.

INTRODUCTION

Bak, Tang & Wiesenfeld (1988) introduced the concept of self-organized criticality in terms of a cellular-automaton, sandpile model. A square grid of boxes was considered and at each time step a particle was randomly dropped into a box. When a box accumulated four particles they were redistributed to the four adjacent boxes, or in the case of edge boxes lost from the grid. Redistributions could lead to further instabilities with, at each time step, the possibility of avalanches of particles being lost from the grid. An avalanche is defined as the number of particles lost from the grid in a time step. Extensive numerical studies of this cellular-automata model were carried out by Kadanoff *et al.* (1989) and the non-cumulative frequency–size distribution of avalanches was found to have a fractal distribution with a slope near unity. The model behaviour was considered as an analogue to actual sandpiles and a variety of experiments were carried out on the avalanche statistics of granular piles. In some cases reasonably good fractal statistics were found and in other cases they were not (Nagel 1992).

A system is said to be in a state of self-organized criticality if when perturbed it returns to a state of marginal stability. The input is steady, but the output is a series of 'avalanches' that obey fractal statistics. The system oscillates about the point of marginal stability. Distributed seismicity on the margin between two tectonic plates (e.g. California) is taken as an example of self-organized criticality (Bak & Tang 1989). The relative motion of the plates provides a continuous input of energy; this stored elastic energy is lost in earthquakes that have a universal power-law frequency–size distribution (Turcotte 1992).

Burridge & Knopoff (1967) introduced the coupled slider-block model as an analogue for earthquakes. The slider blocks are pulled over a surface by springs attached to a constant-velocity driver plate and are also attached to each other by springs. If the static friction is greater than the slipping (dynamic) friction, stick-slip behaviour is found. Huang & Turcotte (1990) showed that two slider blocks exhibit classical low-dimensional chaotic behaviour as long as there is any asymmetry in the system. Carlson & Langer (1989) considered large numbers of slider blocks and found that the non-cumulative frequency–size statistics of smaller slip events are fractal with a slope near unity. Thus multiple slider-block models are also considered to be an example of self-organized criticality. A wider variety of slider-block models have been proposed and studied; these have been reviewed by Carlson, Langer & Shaw (1994).

In the standard model, blocks of mass m are pulled over a surface with a driver plate moving at a constant velocity V. The blocks are connected to the driver plate by driver springs, spring constant k_d, and to each other by connector springs, spring constant k_c. The blocks have a frictional interaction with the surface, and the simplest friction model introduces a static ($v=0$) friction force F_{static} and a dynamic ($|v|>0$) friction force F_{dynamic}. One can see that these friction coefficients introduce a strong non-linearity into the model. If $F_{\text{static}} > F_{\text{dynamic}}$, stick-slip behaviour is observed.

Large slip events in multiple slider-block models require the simultaneous solution of coupled equations of motion for all the moving blocks. Solutions are greatly simplified if only one block is allowed to slip at a time, that is the system is treated as a cellular automaton. Results obtained using the cellular-automaton approach differ little from those that consider the simultaneous slip of multiple blocks. Huang, Narkounskaia & Turcotte (1992) showed that the applicable equations could be rescaled to eliminate the dynamic friction and that generalized results could be obtained that included conservation of energy, that is zero dynamic friction. Rundle *et al.* (1995) found that the block energy distribution is a generalized Maxwell–Boltzmann (exponential) distribution in the limit where the model approaches the mean field with small but non-zero fluctuations. This exponential distribution of energy is still associated with power-law frequency–size statistics for slip events.

There are many similarities between the behaviour of slider-block models and distributed seismicity (Main 1996). One of the most interesting is the universal applicability of the Gutenberg–Richter frequency–magnitude relation

$$\log N = a - bM\,, \tag{1}$$

where N is the number of earthquakes in a specified time interval and region with magnitude greater than M and the b value is nearly constant (typically in the range $b=0.9\pm0.2$). It is easily shown that the b value is simply related to the fractal dimension D, $N\sim r^{-D}$, with r the earthquake rupture length, taking $D=2b$ (Turcotte 1992). The a value is a measure of the intensity of regional seismicity and has considerable variability.

The objective of this paper is to develop a better understanding of the statistical mechanics of stick-slip systems using a modified slider-block model. In order to do this we consider a 2-D array of slider blocks which are connected to each other by springs and which interact frictionally with a surface; no driver plate is included. We assume that the dynamic friction is zero so that energy is conserved. The initial distribution of energies in the springs is prescribed and the ratio of the mean energy per spring to the prescribed static friction is the only parameter in the problem. For sufficiently high energies most of the blocks are slipping; for low energies most of the blocks are 'stuck'. Our objective is to study the statistical distribution of energies in the springs and the statistical distribution of moving patches. Although our approach is highly idealized, we believe that it contributes to a basic understanding of distributed seismicity.

It is appropriate to note that the statistical mechanics of thermodynamic systems such as an ideal gas are also highly idealized and energy conserving. The Maxwell–Boltzmann distribution of velocities is a universal output. This statistical mechanics approach has been extended to a wide variety of irreversible processes with energy dissipation. An example would be the laminar viscous flow of an ideal gas through a pipe. The energy input is the pressure gradient and the energy loss is the frictional heating of the gas. Nevertheless, despite the energy dissipation, the molecular velocities have a Maxwell–Boltzmann distribution to a good approximation (Chapman & Cowling 1960). We suggest that our simple model (as well as related simple models such as site percolation) is related to distributed seismicity in much the same way that viscous flows are related to ideal thermodynamic behaviour.

MODEL

In this paper we consider a 2-D square array of slider blocks. Each block of mass m is connected to its four neighbours with springs (spring constant k_c) and is confined to move in the x-direction. The model is illustrated in Fig. 1. We assume zero dynamic friction and prescribe the static friction F_{static} between a block and the surface over which it is sliding. The initial total energy in the system is also prescribed. Because the dynamic friction is zero, the total energy in the system is preserved. Some blocks are unstable at a given time and are free to slip; these are considered to be equivalent to active faults. At this time other faults are stable and do not slip; these are considered to be equivalent to inactive faults.

We consider square arrays of slider blocks and a particular slider block is designated by the subscript i (position in the x-direction) and j (position in the y-direction). The net force on block (i, j), $F_{i,j}$, is given by

$$F_{i,j} = k_c(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} - 4x_{i,j})\,, \tag{2}$$

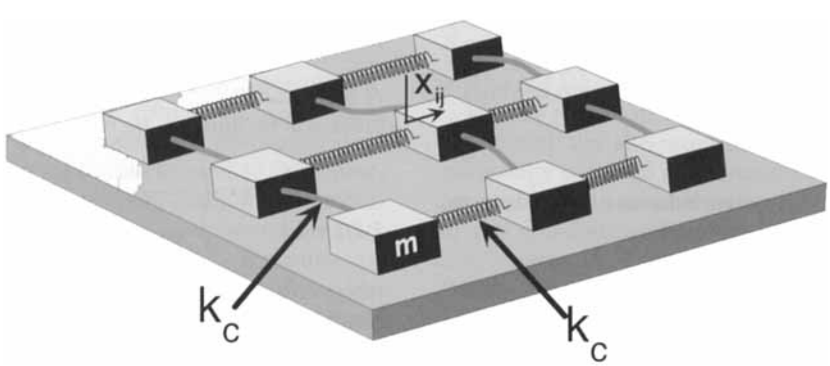

Figure 1. Illustration of the slider-block model. A square array of blocks of mass m are connected to adjacent blocks with either leaf or coil springs with spring constants k. The blocks are free to slip only in the x-direction and the displacement of block (i, j) is $x_{i,j}$.

where $x_{i,j}$ is the displacement of block (i, j) in the x-direction from the initial undisturbed position. A block is unstable if $F_{i,j} > F_{\rm static}$ and is stable if $F_{i,j} < F_{\rm static}$.

In order to simulate earthquakes we assume only one block is slipping at a given time step. Sequential sweeps through the lattice are carried out. The results given in this paper utilize a 'checker-board' algorithm: during one half of a sweep we check the 'black sites' on the checker-board and then during the second half we check the 'white sites'. At a given time step the net force on the chosen block (i, j), $F_{i,j}$ is determined from (2). If the block is stable ($F_{i,j} < F_{\rm static}$) it does not move; if the block is unstable ($F_{i,j} > F_{\rm static}$) it is allowed to slip. The motion of the single slipping block during an update is given by

$$m\frac{d^2x_{i,j}}{dt^2}=F_{i,j}\,. \tag{3}$$

Noting that the four adjacent blocks remain fixed, the slipping block executes half of a harmonic cycle and sticks when the velocity is again zero. The change in the position of the block (i, j), $\Delta x_{i,j}$, during a time step is related to the initial net force on the block $F_{i,j0}$ by

$$\Delta x_{i,j}=\frac{F_{i,j0}}{2k_{\rm c}}\,, \tag{4}$$

and the block considered is given this displacement.

At $t=0$ the blocks are given a random distribution of displacements; the resulting energy in spring s is E_s. The mean energy in the springs at $t=0$ is $\langle E_s\rangle$. Since no energy is dissipated by dynamic friction (the dynamic friction is taken to be zero), energy is conserved and $\langle E_s\rangle$ is a constant, independent of time. When a block slips the total energy in the four attached springs is the same after the slip event as it was before the slip event. It is convenient to introduce the non-dimensional energy parameter μ:

$$\mu=\frac{k_{\rm c}\langle E_s\rangle}{F^2_{\rm static}}\,. \tag{5}$$

This is the only parameter in the problem. If μ is large very few of the blocks will stick; if μ is small a large fraction of the blocks will stick. We further introduce the non-dimensional variables

$$\bar{F}_{i,j}=\frac{F_{i,j}}{F_{\rm static}}\,,\qquad \bar{x}_{i,j}=\frac{k_{\rm c}x_{i,j}}{F_{\rm static}}\,,\qquad \bar{E}_s=\frac{k_{\rm c}E_s}{F^2_{\rm static}}\,. \tag{6}$$

The force balance (2) becomes

$$\bar{F}_{i,j}=\bar{x}_{i,j-1}+\bar{x}_{i,j+1}+\bar{x}_{i-1,j}+\bar{x}_{i+1,j}-4\bar{x}_{i,j}\,, \tag{7}$$

and the stability condition for a block is $\bar{F}_{i,j}<1$. The displacement of a block during a slip event from (4) becomes

$$\Delta\bar{x}_{i,j}=\frac{1}{2}\bar{F}_{i,j0}\,. \tag{8}$$

In order to determine how this system behaves it is necessary to carry out a series of numerical simulations.

SIMULATIONS

We have carried out a series of simulations on square arrays of up to 3000×3000 blocks using a Pentium 120 machine running Linux OS. Springs on the boundaries of the array are attached to fixed walls. Various values of the energy parameter μ have been considered. In each simulation the fraction of blocks slipping is determined as a function of time. The distribution of energies in the springs was also determined at various times. Various initial distributions of block displacements were considered. The governing equations (7) and (8), along with the stability criteria, were then used with the checker-board algorithm to study the evolution of the system. The system was allowed to evolve until the statistical distribution of energies reached a steady state. The required time was less than 20 sweeps except for initial energies μ smaller than 0.09. Below that value the system evolved very slowly. For all initial energies above $\mu=0.09$ excellent agreement of the final distribution of energies in the springs with a modified Maxwell–Boltzmann distribution was found.

The modified Maxwell–Boltzmann probability distribution function for spring energies in one dimension is given by

$$p(\bar{E}_s)=\frac{\exp\left(-\dfrac{\bar{E}_s}{2\mu}\right)}{\sqrt{2\pi\mu\bar{E}_s}}\,, \tag{9}$$

where $p(\bar{E}_s)$ is the probability density. A typical distribution of non-dimensional spring energies for a simulation with $\mu=1$ is given in Fig. 2. The results are seen to be in in agreement with the predicted value from (9). The system evolved to this equilibrium distribution independent of the initial distribution of energies chosen.

In the equilibrium state the fraction of blocks sliding is only a function of the non-dimensional energy parameter μ. This dependence is shown in Fig. 3. We will now relate the fraction of blocks sliding to the distribution of spring energies given in (9). The corresponding probability distribution function for the forces on the springs $p(\bar{F}_s)$ is

$$p(\bar{F}_s)=\frac{\exp\left(-\dfrac{\bar{F}_s^2}{4\mu}\right)}{2\sqrt{\pi\mu}}\,. \tag{10}$$

This is a Gaussian distribution and is equivalent to the Maxwell–Boltzmann distribution of molecular velocities in one dimension. However, the slip condition for a block is determined by the statistical distribution of forces on the blocks. From (7) it is seen that the random force on a block is the sum of four random forces on the neighbouring springs, and the Gaussian distribution of forces on the blocks $p(\bar{F}_{i,j})$ is

$$p(\bar{F}_{i,j})=\frac{\exp\left(-\dfrac{\bar{F}_{i,j}^2}{32\mu}\right)}{4\sqrt{2\pi\mu}}\,. \tag{11}$$

A block can slip if $|\bar{F}_{ij}|>1$. Using (11) the probability that a block will be slipping, P_s, is

$$P_s(\mu)=2\int_1^\infty\frac{\exp\left(-\dfrac{\bar{F}_{i,j}^2}{32\mu}\right)}{4\sqrt{2\pi\mu}}\,d\bar{F}_{i,j}=\operatorname{erfc}\left(\frac{1}{\sqrt{32\mu}}\right), \tag{12}$$

where erfc is the tabulated complementary error function. This result is compared with the numerical experiments in Fig. 3 for various values of the non-dimensional energy μ. Excellent agreement is found; it should be noted once again that the solid line is a theoretical prediction rather than an empirical correlation.

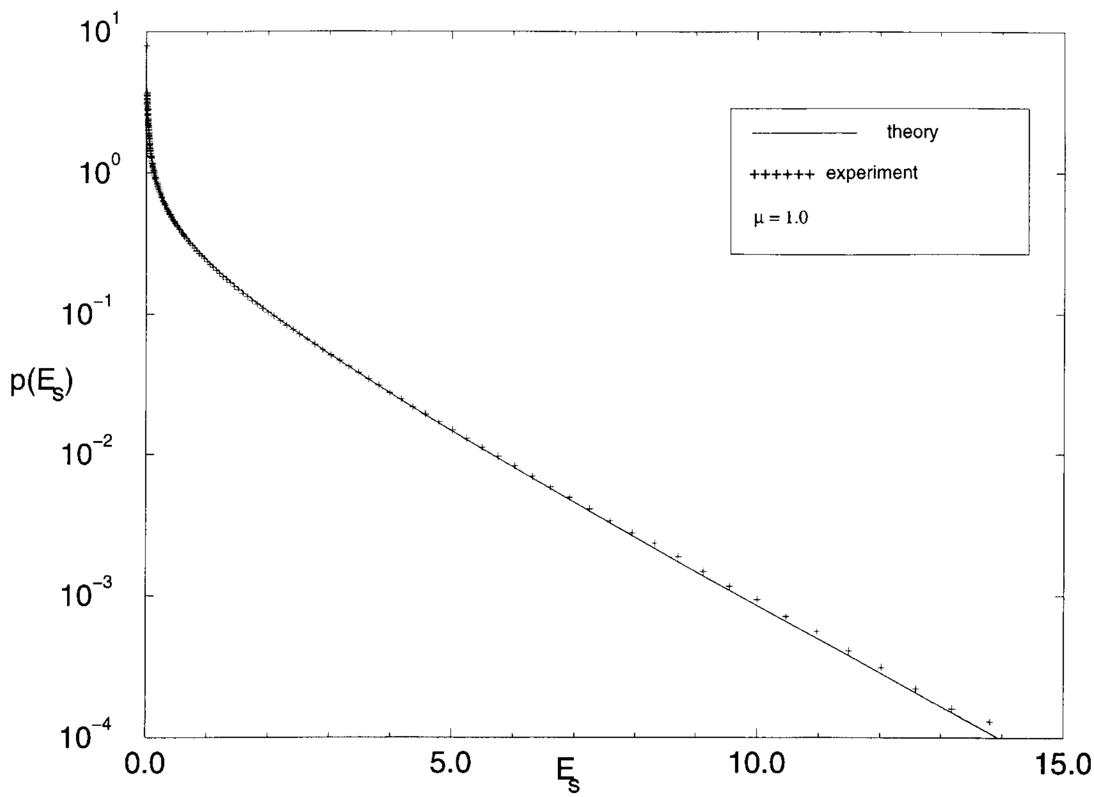

Figure 2. The probability distribution $p(E_s)$ of the non-dimensional energies E_s in the springs of a multiple slider-block model. The crosses are the result for a 2000×2000 array of slider blocks with $\mu = 1$. The solid line is the modified Maxwell–Boltzmann distribution of energies given in (9).

As the fraction of slipping blocks increases with increasing values of μ, a continuous path of slipping blocks across the 2-D array is eventually established. In terms of fracture mechanics this path is analogous to a throughgoing fracture. As the stress on a brittle material is increased the number of microcracks increases, and eventually the microcracks coalesce to form a throughgoing rupture. In terms of regional seismicity we consider a continuous path of slipping blocks to be analogous to the tectonic disruption of the Earth's crust at a plate boundary. The creation of a continuous path of slipping blocks also suggests a close analogy between our slider-block problem and the site-percolation problem.

SITE-PERCOLATION MODEL

The site-percolation model has been studied extensively in the physics literature as a simple example of a phase change (Stauffer & Aharony 1992). In the 2-D site-percolation problem a square matrix of boxes is considered, where each box is either permeable or impermeable. The probability that a box is permeable, p, is specified and is used to determine whether a specific box is permeable. The question is whether the square array of boxes is permeable or impermeable (the array is considered to be permeable if there is a continuous path of permeable boxes from one side of the array to the other). This is clearly a statistical problem because the actual distribution of permeable and impermeable boxes is random. For a specified value of the microscopic probability, p, there is a macroscopic probability, P, that the $n \times n$ array of boxes is permeable. For large arrays it is found that P is very small if $0 < p < p_0$, where p_0 is the critical probability for the percolation threshold and P is near unity for $p_0 < p < 1$. There is thus a critical value of p, p_0, for the onset of flow through the array of boxes. For a 2-D $n \times n$ array with large n, numerical simulations find that the critical permeability is $p_0 = 0.59275$ (Stauffer & Aharony 1992). Those boxes that are part of the permeable cluster that crosses the array are known as the percolation backbone.

The percolation threshold in the site-percolation model is known to be a classic example of a critical point (Stauffer & Aharony 1992). Power-law scalings are found to be valid in the vicinity of this critical point. One example is the number–size distribution of clusters at the critical point. The number of clusters N_s with area M_s is given in Fig. 4 for a 3000×3000 box array. Good agreement with the fractal relation

$$N \sim A^{D/2} \tag{13}$$

is obtained taking $D = 1.0$.

We now make a direct comparison between our slider-block model and the site-percolation model. Just as the critical probability p_0 in the site-percolation model corresponds to a permeable path across the array, we can obtain a critical value of the non-dimensional energy μ, μ_0, for which there is a continuous path of slipping blocks across the slider-block array. We further suggest that this is also a critical point. We find that a path of slipping blocks across the slider-block array is obtained when the non-dimensional energy parameter $\mu_0 = 0.213$. We take this to be a critical value of the energy parameter. The corresponding fraction of slipping blocks (from Fig. 3) is $P_s = 0.583$. This value can be compared with the critical point of

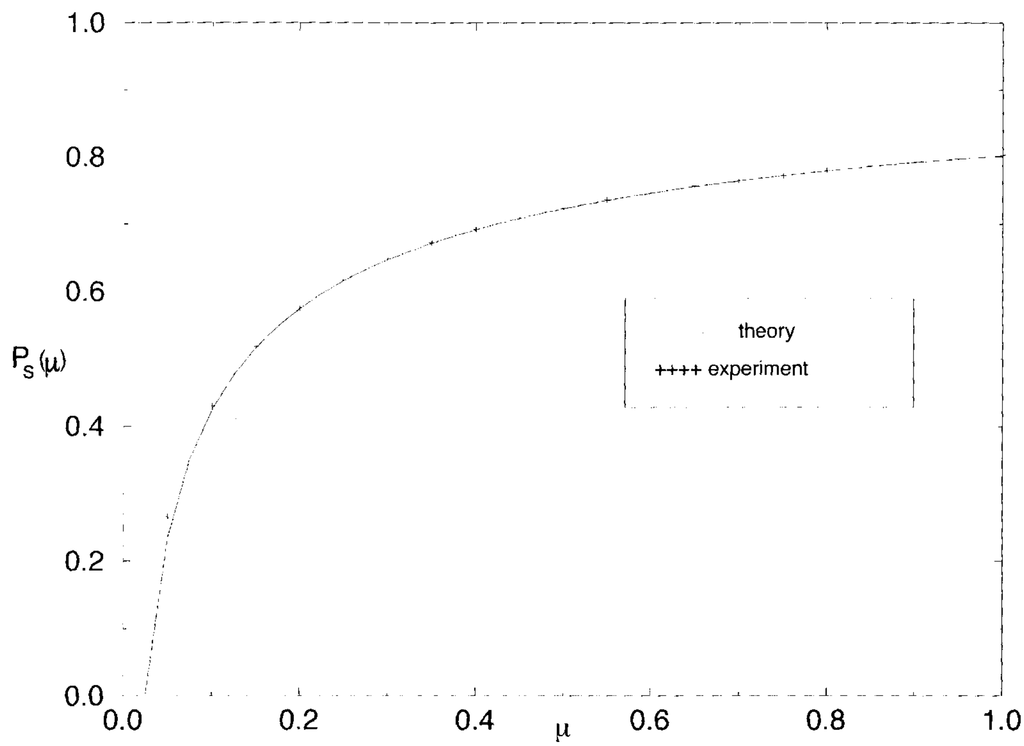

Figure 3. The fraction of the blocks that are slipping $P_s(\mu)$ is given as a function of the mean energy μ. The crosses are results for a 1000×1000 array of slider blocks and the solid line is the prediction from (12) based on the Gaussian distribution of forces given in (11).

the site-percolation model, where the critical probability is $p_0 = 0.59275$. The close agreement between critical probability for the site-percolation model and the critical fraction of slipping blocks suggests a close analogy between the two problems. A typical slider-block configuration with a continuous path of slipping blocks across the array is shown in Fig. 5. This is essentially identical to the distribution of permeable boxes in the site-percolation model at the critical point.

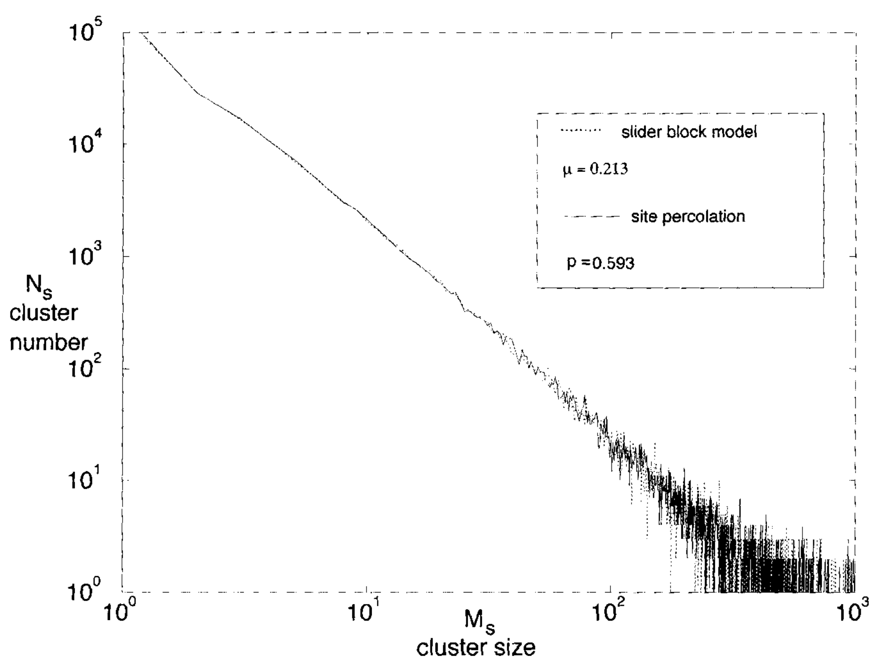

Figure 4. Number of clusters N_s of size M_s as a function of M_s. The solid line is the distribution of clusters for the site percolation model for a 3000×3000 array with the critical site-percolation probability $p = 0.5927$. The dashed line is the distribution of sliding clusters of blocks on our 3000×3000 array of slider blocks at the critical point $\mu = 0.213$.

We have also determined the number–size distribution of slipping clusters of blocks at the critical non-dimensional energy $\mu_0 = 0.213$. The result is given in Fig. 4 and is seen to be virtually indistinguishable from the cluster distribution for the site-percolation problem at the critical probability $p_0 = 0.59275$. Our results strongly suggest that the occurrence of a continuous path of slipping blocks is a critical point for this type of slider-block model.

DISCUSSION

We have obtained several interesting results. The first is the remarkable similarity between the slider-block model and the site percolation model. This suggests that both are part of a universality class (Bruce & Wallace 1989) that may underlie a variety of practical applications. The second interesting result is the evolution of the model to a modified Maxwell–Boltzmann distribution of energies. The results become independent of initial conditions and the single control parameter is the non-dimensional energy of the system μ. This control parameter can be adjusted so that critical-point behaviour is obtained. The critical point corresponds to a continuous path of slipping blocks across the array. This can be considered to be analogous to the nucleation and propagation of a rupture in a brittle material. Hirata, Satoh & Ito (1987) studied the fracture of a pristine granite block under an applied stress. The stress on the block is analogous to the energy in our slider-block problem. Hirata *et al.* (1987) found that the initial microfractures in the granite were uncorrelated but as the number increased and a throughgoing rupture ensued, the cracks obeyed fractal statistics. The initiation of a throughgoing rupture in granite appears to be quite analogous to the throughgoing path of slipping blocks in our slider-block model.

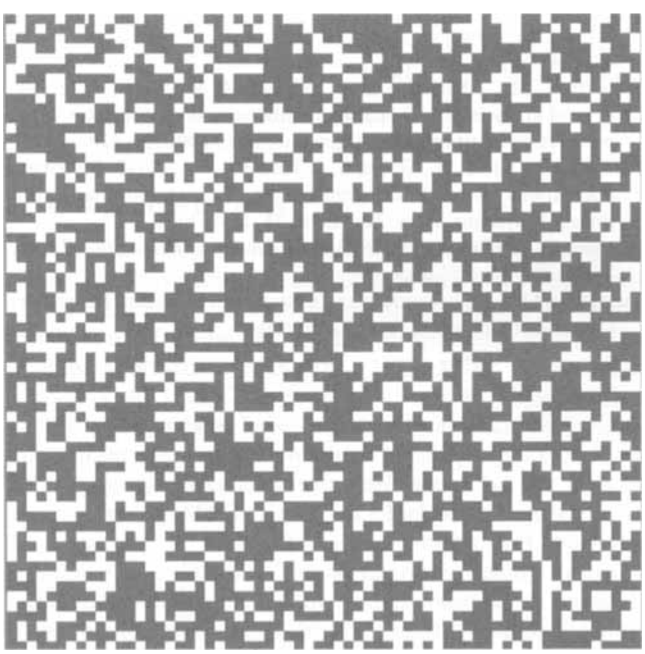

Figure 5. Illustration of a typical configuration of sliding blocks at the critical point $\mu = 0.213$ for a 64×64 array. White blocks are stuck and black blocks are sliding. A continuous path of sliding blocks across the array is present.

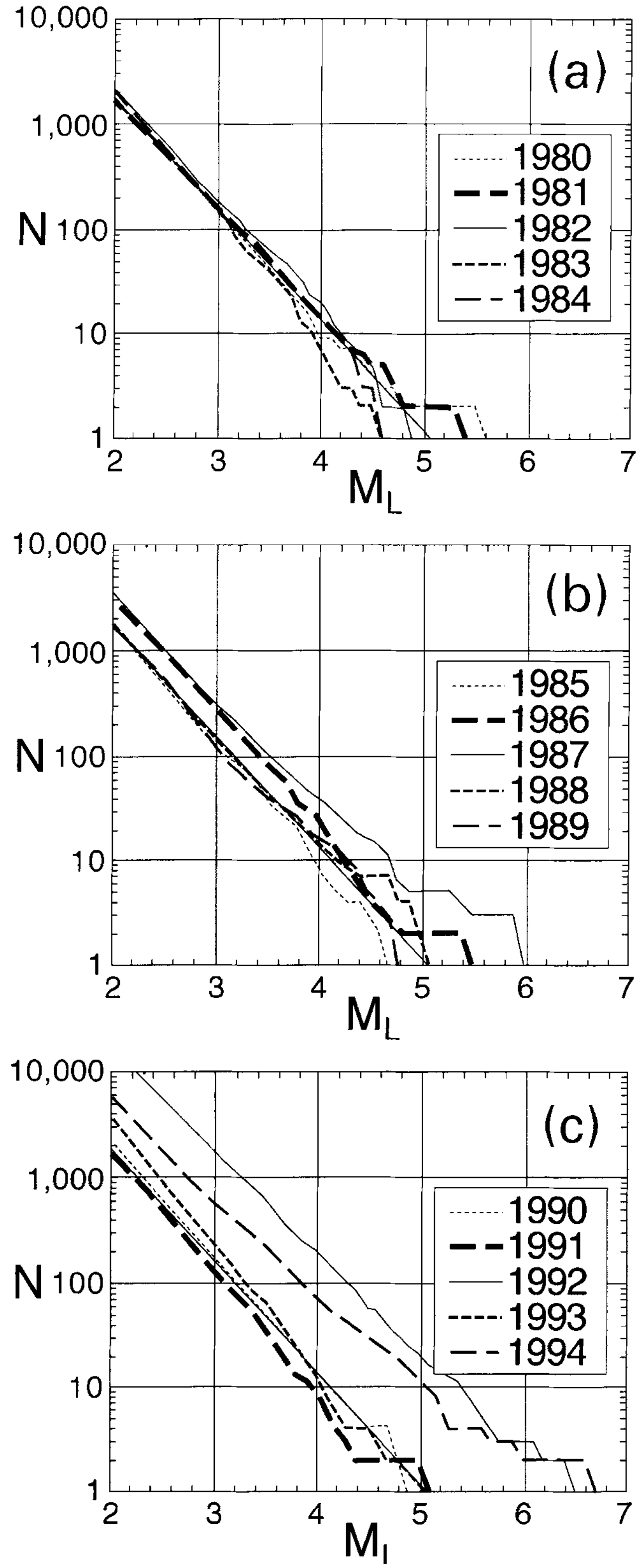

Figure 6. The cumulative number of earthquakes N with magnitude greater than M_L for each year between 1980 and 1994 is given as a function of M_L; the region considered is southern California. The straight-line correlation is the Gutenberg–Richter relation (1) with $a = 4.3$ and $b = 1.06$.

There are also indications of strong similarities between our slider-block model and distributed seismicity. The tectonic disruption at a plate boundary is similar to the path of slipping blocks in our model at the critical point. Observational confirmation of this comes from the studies of earthquake epicentres by Robertson *et al.* (1995). These authors showed for a series of aftershock sequences (and for the distribution of earthquakes in the Parkfield section of the San Andreas fault) that the spatial distribution is fractal. The fractal dimensions were near $D=2$ even though the array of earthquakes is not planar. The fractal dimension of the percolation backbone of a 3-D site-percolation model is also near two. This led Robertson *et al.* (1995) to suggest that distributed seismicity took the form of a percolation backbone of a 3-D percolating cluster, in accord with our results.

Further support for this statistical physics approach to seismicity comes from the frequency–magnitude distribution of earthquakes in southern California. The cumulative number of earthquakes N with magnitude greater than M_L for each year between 1980 and 1994 is given as a function of M_l in Fig. 6 (Newman, Turcotte & Gabrielov 1995). An excellent correlation with the Guttenberg–Richter relation (1) is obtained taking $a=4.3$ and $b=1.06$. It is seen that the intensity of seismicity, the a value, changes little from year to year. This suggests stationarity (Main 1996). The major deviations in 1987, 1993 and 1994 can be attributed to the aftershocks of the Whittier Narrows, Landers and Northridge earthquakes, respectively. The constancy of the low-level seismicity in time is remarkable considering the extent of the spatial variability. Again this is suggestive of a system which is residing at or near a critical point or a point of marginal stability (Carlson & Langer 1989).

The applicability of the model to distributed seismicity can certainly be questioned. The model conserves energy whereas earthquakes are dissipative. During an earthquake energy is lost both to frictional heating and in radiated seismic energy. The source of energy is the movement of the tectonic plates. Nevertheless, the similarities of the statistical behaviour of seismicity and the model behaviour, and the similarities between our model and the site-percolation model provide evidence for a universality to both the model and the natural phenomena.

ACKNOWLEDGMENTS

It is a pleasure to thank Jim Sethna, John Rundle and Bill Newman for useful discussions and comments. This research was partially supported under NSF grant# EAR 94-23818; AG was also partly supported by the US Army Research Office through the Mathematical Sciences Institute of Cornell University, Contract DAAL03-91-C-0027.

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