Solutions to Exam 2

Note Title 11/3/2005

1. a) $$\frac{2x+7}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$2x+7 = A(x+1) + B(x-4)$$
$$= (A+B)x + (A-4B)$$

$$\begin{aligned} A+B &= 2 \\ A-4B &= 7 \end{aligned} \qquad \begin{cases} A=3 \\ B=-1 \end{cases}$$

$$I = 3\,\mathrm{Ln}|x-4| - \mathrm{Ln}|x+1| + C$$

b) $$\frac{2x+7}{(x-1)^2+4} = \frac{2[(x-1)+1]+7}{(x-1)^2+2^2}$$

$$= \frac{2(x-1)}{(x-1)^2+2^2} + \frac{9}{(x-1)^2+2^2}$$

$$I = \mathrm{Ln}[x^2-2x+5] + \frac{9}{2}\mathrm{Tan}^{-1}\frac{(x-1)}{2} + C$$

c) $u = \ln(x+2)$ $\quad dv = 2x\,dx$

$du = \frac{1}{x+2}\,dx$ $\quad v = x^2$

$$I = x^2 \ln(x+2) - \int x^2 \frac{1}{x+2}\,dx$$

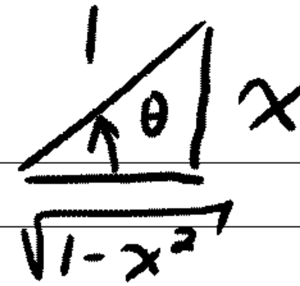
$$\begin{array}{r}
x - 2 \\
x+2 \,\big)\, \overline{x^2 \qquad\qquad} \\
\underline{x^2 + 2x} \\
-2x \qquad \\
\underline{-2x - 4} \\
4
\end{array}$$

$$\int \frac{x^2}{x+2}\,dx = \int x - 2 + \frac{4}{x+2}\,dx$$

$$= \frac{1}{2}x^2 - 2x + 4\ln|x+2| + C$$

$$I = x^2 \ln(x+2) - \frac{1}{2}x^2 + 2x - 4\ln(x+2) + C$$

d) $x = \sin\theta$



$$I = \int \frac{\left(\sqrt{1-x^2}\right)^3}{x^6}\, dx$$

$$= \int \frac{\cos^3\theta}{\sin^6\theta} \cos\theta \, d\theta$$

$$= \int \left(\frac{\cos\theta}{\sin\theta}\right)^4 \frac{1}{\sin^2\theta}\, d\theta$$

$$= \int \cot^4\theta \csc^2\theta \, d\theta$$

$u = \cot\theta \qquad du = -\csc^2\theta \, d\theta$

$$I = -\int u^4 \, du = -\frac{1}{5} u^5 + C$$

$$= -\frac{1}{5}\cot^5\theta + C = -\frac{1}{5}\left(\frac{\sqrt{1-x^2}}{x}\right)^5 + C$$

e) $u = (2x-3)$ $\quad du = 2\,dx$

$x = \frac{1}{2}(u+3)$

$$I = \int x(2x-3)^5\,dx$$

$$= \int \left[\tfrac{1}{2}(u+3)\right] u^5 \left(\tfrac{1}{2}du\right)$$

$$= \frac{1}{4}\int u^6 + 3u^5\,du$$

$$= \frac{1}{4}\left[\frac{1}{7}u^7 + \frac{3}{6}u^6\right] + C$$

$$= \frac{1}{28}(2x-3)^7 + \frac{1}{8}(2x-3)^6 + C$$

2.a) $$\lim_{x\to 0+} x(\ln x)^2 = \lim_{x\to 0+} \frac{(\ln x)^2}{\left(\frac{1}{x}\right)} =$$

$$= \lim_{x\to 0+} \frac{2(\ln x)\frac{1}{x}}{\left(-\frac{1}{x^2}\right)} = \lim_{x\to 0+} \frac{2\ln x}{\left(-\frac{1}{x}\right)}$$

$$= \lim_{x \to 0^+} \frac{2 \frac{1}{x}}{\left(\frac{1}{x^2}\right)} = \lim_{x \to 0^+} 2x = 0$$

b) Let $f(x) = (\ln x)^{1/x}$.

$$\lim_{x \to \infty} \ln f(x) = \lim_{x \to \infty} \frac{1}{x} \ln(\ln x)$$

$$= \lim_{x \to \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = 0.$$

So $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} e^{\ln f(x)} = e^0 = \underline{1}$

3. $$\int \frac{dy}{y^2+1} = \int 2x \, dx$$

$$\tan^{-1} y = x^2 + C$$

$y(0) = 1$: $\tan^{-1} 1 = 0^2 + C$

$$\frac{\pi}{4} = C$$

So $\operatorname{Tan}^{-1} y = x^2 + \frac{\pi}{4}$

$$y = \operatorname{Tan}\left(x^2 + \frac{\pi}{4}\right)$$

4. $T(1) = 200e^{-K\cdot 1} = 100$

$$e^{-K} = \frac{1}{2}$$

$$\ln e^{-K} = \ln \frac{1}{2}$$

$$-K = \ln \frac{1}{2} = -\ln 2$$

So $K = \ln 2$.

$$T(3) = 200e^{-(\ln 2)3} = 200e^{\ln 2^{-3}} = \frac{200}{8}$$

$$= 25$$

<u>OR</u> $T(3) = 200e^{-3K} = 200\left(\underbrace{e^{-K}}_{=\frac{1}{2}}\right)^3$

$$= 200\left(\frac{1}{2}\right)^3 = 25.$$