

Suggestions for Functional Analysis Basics (FAB) Seminar

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Functional analysis is a central topic in analysis at an advanced level and is at the foundation of many parts of analysis, including differential equations (both ordinary and partial), mathematical physics, numerical analysis, signal processing and control theory in engineering, and complex and real analysis, in addition to the subjects in the areas of Modern Analysis and Geometry that are primarily functional analysis: operator theory, C^* and operator algebras, and non-commutative geometry, topology, and probability. Functional analysis extends the theory of linear algebra (over the real or complex fields) to infinite dimensional spaces and thereby requires the conscious inclusion of ideas related to continuity to get problems and theorems applicable to a broad variety of important areas of applications and mathematics.

The necessary background for success in my courses at the level of Math 54600 is one or two semesters of analysis (e.g. a graduate course in analysis like Math 50400 or Math 54400) and an undergraduate course in linear algebra.

Recommended reading: *A Guide to Functional Analysis* by Steven G. Krantz, MAA, 2013 (ISBN: 978-0-88385-357-3), about \$25 (PDF), \$50 (hardcover)), or online access through Purdue Library. The material covered in Math 54600 is approximately Chapters 1, 2, 3, 4, 8, and 9 from Krantz's book with a little of Chapters 5 and 6 added. Of course, there are some differences depending on the instructor.

The following books cover most of the material that is central, in my view, for a course like Math 54600, but they each have different perspectives than mine on learning the subject. Nevertheless, they are useful references for your seminar. The books marked with an asterisk(*) will be on reserve for the semester in the Math library.

J. B. Conway	<i>A Course in Functional Analysis*</i>
J. B. Conway	<i>A Course in Operator Theory*</i>
B. D. MacCluer	<i>Elementary Functional Analysis</i>
R. E. Megginson	<i>Introduction to Banach Space Theory*</i>
A. Taylor and D. Lay	<i>Introduction to Functional Analysis*</i>
P. R. Halmos	<i>Introduction to Hilbert Space . . .*</i>
P. R. Halmos	<i>Hilbert Space Problem Book</i>
W. Rudin	<i>Real and Complex Analysis</i>
W. Rudin	<i>Functional Analysis</i>

The following books are less central, in my view, for the material of this course, but they might be useful references. Except for Baggett and Groetsch, they are more advanced than those listed above. None of these books is on reserve in the library.

L. W. Baggett	<i>Functional Analysis: A Primer</i>
S. K. Berberian	<i>Lectures in Functional Analysis</i>
C. W. Groetsch	<i>Elements of Applicable Functional Analysis</i>
Dunford and Schwartz	<i>Linear Operators (3 Volumes)</i>
Riesz and Sz. Nagy	<i>Functional Analysis</i>
K. Yosida	<i>Functional Analysis</i>

Topics appropriate for your seminar

- Definitions, examples, and elementary properties
- Linear functionals and convexity: extension and separation theorems, weak topologies, Krein-Milman theorem
- Duality
- Completeness: uniform boundedness, open mapping, and closed graph theorems
- Hilbert spaces and generalized Fourier series
- Operators: spectrum, spectral mapping theorem, Riesz functional calculus
- Compact Operators
- Spectral theorem for self-adjoint and normal operators on Hilbert space
- Unbounded operators (if time permits)

Some philosophy

3 Keys to Learning Mathematics

1. Work lots of problems.
2. Memorize definitions and the statements of major theorems.
3. Work lots more problems.

Ordinarily, after mathematicians and scientists complete research on a topic, they write up their results and submit them for publication to an appropriate journal. To be accepted for publication, the work must be interesting, correct, and reasonably well written. By hypothesis, all homework problems on the list on the webpage above are interesting. Your job is to tackle some of the problems that interest you and be prepared to present them to someone else with enough details in the proof to be convincing.

Publication of jointly authored research in mathematics is frequent and acceptable: the authors do the research together and write up a single account of their work for submission with all their names as authors.

There is an unwritten code of ethics governing joint publication. Whether work discussed with another mathematician is “joint” or not is determined by the individuals involved by mutual consent. One standard is that the work is jointly authored if the contribution of each author is “significant”. It is unethical to submit jointly authored work as yours alone. It is equally unethical to include an author who has contributed nothing. Assignment of credit by professional communities for jointly authored work is always problematic and ranges from the generous: each author gets full credit for the whole paper; to the blatantly unfair: you get no credit, the other author gets full credit. In the seminar, it would be appropriate for you to work together and giving credit to your co-authors if you present the work to others.