Symposium on Analysis and PDEs

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Four lectures on the theory of Fast Diffusion

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The minicourse is aimed at presenting some of the recent progress in the mathematical theory of nonlinear diffusion processes by focusing on the model called the fast diffusion equation.

Lesson 1. Introduction to Linear and Nonlinear Diffusion. Main models in problems of viscous fluids, phase change, water infiltration, population dynamics, and plasma physics.

The Fast Diffusion Equation and the Porous Medium Equation. Main features. Slow and fast propagation.

Lesson 2. Existence of different types of solutions. Well-posedness. Existence in optimal classes of data. Cases of non-uniqueness and non-existence. The role of critical exponents.

Regularity of the solutions: smoothing effects and continuity.

Lesson 3. Asymptotic behaviour of the solutions for large time. Extinction. Special Selfsimilar solutions

Lesson 4. The geometrical models. The evolution flows: Yamabe ($n \geq 3$) and Ricci ($n = 2$).

New lines of research: p-Laplacians and fast diffusion

References: A large part of the material is taken from the following monograph

J. L. Vázquez, *Smoothing and decay estimates for nonlinear diffusion equations*, vol. 33 of Oxford Lecture Notes in Maths. and its Applications, Oxford Univ. Press, 2006.