A problem in potential theory arising in biology

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Let K_0 and K_1 be two bounded, disjoint convex sets in $R^n, n \geq 3$, and u the equilibrium potential, that is the harmonic function in $R^n \backslash \{K_0 \cup K_1\}$ such that u has boundary values 1 on $K_1 \cup K_2$ and $u(x) \to 0, x \to \infty$. Denote

$$r_j = r_j(K_0, K_1) = \int_{\partial K_j} \frac{\partial u}{\partial n} ds, \quad j = 0, 1,$$

where n is the inner normal and ds is the surface area element. Is it true that each r_j decreases if we move K_0 and K_1 closer to each other?

More precisely: Let $f : R^n \to R^n$ be a distance decreasing homeomorphism, whose restriction on each K_j is an isometry onto the $f(K_j)$. Is it true that $r_j(f(K_0, f(K_1)) \leq r_j(K_0, K_1)$?

Comments. This problem originates in the attempts of biologists to explain why certain animals (like armadillos) group close together when they sleep. Presumably this minimizes the rate of heat loss r_j. It is easy to prove that $r_1 + r_2$ decreases when we move animals K_1 and K_2 closer together, see http://www.math.purdue.edu/ eremenko/dvi/armadillo.pdf and references there.

But each individual animal K_j feels only r_j not the sum. Some condition of convexity type is indeed needed here, think of a kangaroo putting her child in the bag.

The problem is unsolved even when K_0 and K_1 are two balls of different radii.