Equilibrium points of Newtonian potentials

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Consider the potential generated by finitely many positive charges in 3-space:

$$u(x) = \sum_{k=1}^{n} \frac{a_k}{|x - x_k|}, \quad a_k > 0.$$

A point x is an equilibrium point if $grad\, u(x) = 0$.

Question 1. Is the set of equilibrium points always finite? (It can be infinite if we allow charges a_k of different signs).

Question 2. If the set of equilibrium points is finite, how many points can it contain?

The answer to question 2 is unknown even if there are three points and all $a_k = 1$. J. C. Maxwell stated without proof that the number of equilibrium points is at most $(n-1)^2$. This gives 4 for $n = 3$ which would be best possible. Gabrielov, Novikov and Shapiro proved that for $n = 3$ there cannot be more than 12 equilibrium points.

References

[1] A. Gabrielov,, D. Novikov and B. Shapiro, The mystery of point charges, arXiv:math-ph/0409009

[2] J. C. Maxwell, Treatise on electricity and magnetism, vol. 1, Dover, 1954.