

Fuchsian equations with unitary monodromy

A. Eremenko

November 20, 2021

Consider the set of second order linear Fuchsian equations on the Riemann sphere, with prescribed n singularities at a_1, \dots, a_n and prescribed real exponent differences α_j . Such equations depend on $n - 3$ accessory parameters $\lambda = (\lambda_1, \dots, \lambda_{n-3}) \in \mathbf{C}^{n-3}$. Consider the set

$$E(a_1, \dots, a_n, \alpha_1, \dots, \alpha_n)$$

consisting of those equations whose projective monodromy is a subgroup of $PSU(2)$.

Is it true this set is discrete for all a_j, α_j ?

Is it true that it is finite?

Both statements are known for $n = 4$:

A. Eremenko, Metrics of constant positive curvature with four conic singularities on the sphere, Proc. AMS 148, 9 (2020) 3957–3965; arXiv:1905.02537.