How was the gravity law discovered

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The story with apple was made up by Newtons niece Catherine Burton. She told it to Voltaire and he spread it.

The true story was told by Newton to William Wistone. In 1666 Newton had an idea to check how acceleration of the Moon is related to the acceleration of falling bodies on the Earth surface.

Assuming that the Moon rotates about the Earth on a circular orbit of radius R with period T we obtain its acceleration

$$a_{\text{Moon}} = R(2\pi/T)^2,$$

where the period is

$$T = 27^D 7^h 43^m = 39343^m,$$

the length of month was known since the times immemorial. It was known since Ptolemy that R is approximately 60 times the radius of the Earth. This is done by measuring Moon's parallax. Denoting the radius of the Earth by r, we obtain the formula for the acceleration of the Moon

$$a_{\text{Moon}} = 60r \frac{(2\pi)^2}{39343^2 \cdot 3600} \, m/sec$$

A good value of the Earth radius was a problem for Newton in 1666. He took one minute of the meridian to be 60 statute miles[1].

This number is very imprecise, of course, and gives acceleration $2.39 \cdot 10^{-3}$ So the ratio of acceleration on the Earth surface ($9.8 \, m/sec^2$) to the

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[1] This raises the interesting question why statute mile was defined as it is, 1609 meters Is this related to some very imprecise measurement of the Earth meridian? One minute of the meridian is actually equal to nautical mile, 1853 meters.

acceleration of the Moon he obtained was 4100, which is quite different from the expected $60^2 = 3600$.

Newton concluded that there must be some other force besides gravity which contributes to Moon's acceleration and abandoned the subject.

In 1675 Newton learned about the new measurement of the Earth by Jean Picard. He obtained that 1 minute of the meridian is 69.1 statute miles, which is 1853 meters, close to the modern value. Performing the calculation (I use the modern units, Newton used different ones) he obtained the acceleration

$$a_{\text{Moon}} \approx 2.7 \cdot 10^{-3} \, m \sec.$$

So the ratio of acceleration on the Earth surface to the acceleration of the Moon is $3.63 \cdot 10^3$ which is very close to the ratio of the squares of the distances to the center, 3600.

This convinced Newton that the Inverse Square Law is correct.

Remarks. 1. I already mentioned that Newton had to use different units, for example the Earth circumference was given by Picard as 123249600 Paris feet. More importantly, modern time units were not established at the time of Newton: when measuring the acceleration of a falling body he states what distance the body passes during one oscillation of a pendulum of certain length.

2. As I said this result was obtained in 1675, but Newton did not care to publish it. It was published in Principia only in 1687.

References

[1] Douglas W. MacDougal, Newton's Moon test, in the book: Newton's Gravity, Springer 2012.

[2] I. Newton, The Principia. Mathematical principles of natural philosophy, U. California Press, 1999.