Chasing a U-boat. Solution

We are looking for a parametrized curve, whose speed at every point is k times the distance from the point to the origin. In the original problem $k = 2$. It is natural to use polar coordinates r and ϕ. Let us choose ϕ as the independent variable, then the equation of the curve will be $r = r(\phi)$. The distance from the origin increases at the rate $dr/d\phi$ and the speed is $\sqrt{(r')^2 + r^2}$. (This is calculated from the definition of speed $\sqrt{(x')^2 + (y')^2}$ by substituting $x = r\cos\phi$ and $y = r\sin\phi$. Thus the differential equation is

$$kr' = \sqrt{(r')^2 + r^2}.$$

After squaring, collecting similar temrs and taking square root again, we obtain $\sqrt{(k^2 - 1)}r' = \pm r$. The plus sign should be chosen because the destroyer moves outward. So $r(\phi) = \exp(k^2 - 1)^{-1/2}\phi$.