Wandering domains of entire functions

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Let f be an entire function, which is not of the form $az+b$. Denote by f^n the $n-$th iterate of f, that is $f^n = f \circ \ldots \circ f$ n times. Let F be the maximal open set in the complex plane, where the iterates $\{f^n : n = 0, 1, 2 \ldots,\}$ of f form a normal family.

Suppose that D is a component of F. Consider the set L of all limit functions of the family $\{f^n\}$, and suppose that all these limit functions are constant. There are examples where L is infinite.

Question. *Can L be infinite and bounded?*

Here is a restatement of the question in modern terminology. A component D of the set F is called *wandering* if $f^n(D) \cap f^m(D) = \emptyset$, for all integers $n > m \geq 0$. It is known that all limit functions of the family $\{f^n\}$ in a wandering component D are constant. Furthermore, if the set L of limit functions in a component D of F consists only of constants, and L is infinite, then D is a wandering domain. The question is whether a subdomain $D_1 \subset D$ of a wandering domain can wander on a bounded subset of the plane.

A. Eremenko and M. Lyubich, Examples of entire functions with pathological dynamics, J. London Math. Soc., (2) 36 (1987), 458-468.