GROUP ACTIONS AND THE SINGULAR SET

by

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1. Introduction

Suppose a compact Lie group G is acting on a G–CW complex X. The *singular set* X^S consists of all points in X with non-trivial isotropy subgroups. If the action were free, then the singular set would be empty, otherwise X^S is non-empty. The main purpose of this paper is to show that the *trace* of the action, denoted $\mathrm{tr}(G, X)$, is precisely equal to the trace of the singular set $\mathrm{tr}(G, X^S)$ when the action is not free.

The trace of an action was introduced in [**G**] . It is closely related to the exponent of Browder and Adem, studied in [**B1**], [**B2**], and [**A**]. It is an integer invariant of the action which characterizes a free action of a finite group on a finite complex, and also equals the number of points are in the smallest orbit of an action when G is an elementary abelian p-group.

We first recall briefly the definition of the trace of an action as defined in [**G**]. Then we note that the main results of [**G**] may be expressed in the category of G–CW complexes. The key lemma is established. It gives an obstruction to adjoining an equivariant cell without changing the trace. This implies the main theorem that $\mathrm{tr}(G, X)$ equals $\mathrm{tr}(G, X^S)$.

We apply the key lemma to investigate the relationship between the trace of an action, and the orbit size of the action which we define to be the greatest common divisor of the cardinalities of the orbits. While these two numbers are always equal when an elementary abelian p-group acts on a finite complex (indeed we will show here that these numbers are equal if the group has elementary abelian Sylow p-subgroups for all p), we will show that for cyclic subgroups examples can be found in which the trace and orbit size assume any arbitrary integers subject only to the restriction that the first integer divides the second.

Finally in the last section, we compare the trace and the exponent. We observe they are equal when G is a finite group and X is a finite complex. It is an open question if they agree when X is only finite dimensional. While the trace is defined for every action, the exponent as defined in [**A**] is only defined for finite group actions on finite dimensional complexes. We propose an extension of the definition of the exponent to compact Lie groups.

Many of these results comprise chapter III of [**O**], the Ph.D thesis of Murad Özaydın , Purdue University.

2. The trace of an action

Given a map $f : Y \to X$ between two topological spaces, a *transfer* τ of trace k ($k \in \mathbb{Z}$) for f_* is a graded homomorphism

$$\tau : H_*(X; \mathbb{Z}) \to H_*(Y; \mathbb{Z})$$

so that $f_* \circ \tau(x) = kx$ for all x in $H_*(X;\mathbb{Z})$. The set of all traces associated to transfers for f_* form a subgroup of $\mathbb{Z}$, and we define $\deg(f)$ (degree of f) to be the non-negative generator of this subgroup.

Let $F \to E \to B$ be a fiber bundle. The *trace* of this bundle is the least common multiple of the degrees of projection maps q of the pullback bundles $F \to f^*(E) \xrightarrow{q} Y$ taken over all maps $f : Y \to B$. The *trace of the action* of a group G on a space X is the trace of the Borel construction $X \to X_G \to B_G$, denoted $\operatorname{tr}(G, X)$.

Now a key fact about traces is Proposition 6.6 of [**G**]:

PROPOSITION 1. *Let (G, M) be an action on a compact manifold and suppose B_G has finite type. Then $\operatorname{tr}(G, M) = \operatorname{tr}(p)$ where $p : E \to B$ is a pullback of the universal fibration $p_G : M_G \to B_G$ by any map $f : B \to B_G$ which is a $(\dim M + 2)$-homotopy equivalence. If B is a closed oriented manifold, then $\operatorname{tr}(G, M) = \deg(p)$.*

Now this proposition applies when G is a compact Lie group, since B_G has finite type in this case. We want to extend this result for manifolds M replaced by a finite G–CW complex X. Recall that a G–CW complex is built from equivariant cells of the form $G/K \times e$ (where K, a closed subgroup of G, is the isotropy subgroup of this cell e) with equivariant attaching maps ([**I1**]). A smooth compact manifold with a smooth G action has a G–CW structure [**I2**].

LEMMA 2. *Let G be a compact Lie group and let X be a finite G–CW complex. Then X is a equivariant retract of a compact oriented G-manifold.*

Proof: We may equivariantly imbed X as a subspace of a representation space of G. Then Schultz, [**S**], states that X is an equivariant retract of some open invariant neighborhood V. Using the Haar measure, we can find a smooth invariant map from V to $\mathbb{R}$ which is zero on X and 1 near the frontier of V. The inverse image of a suitably chosen interval gives the compact oriented invariant manifold which we are seeking. □

Hence Proposition 1 is true when M is a finite G–CW complex, since traces are preserved by equivariant retractions.

If $f : B \to B_G$ pulls back a fibre bundle $p : E \to B$ so that $\operatorname{tr}(p) = \operatorname{tr}(G, X)$, then we say that p *realizes* $\operatorname{tr}(G, X)$. When B is a closed oriented m-manifold, then $\operatorname{tr}(G, X) = \deg(p)$. That is the same as saying that the image of $p_* : H_m(E) \to H_m(B) \cong \mathbb{Z}$ is generated by $\operatorname{tr}(G, X)[B] \in H_m(B;\mathbb{Z})$.

We can easily find a closed compact oriented manifold B so that $f : B \to B_G$ is highly connected for G a compact Lie group. It suffices to find a highly connected, closed, oriented manifold E with a free, orientation preserving G-action. Then $B = E/G$ and f is a classifying map for the principal bundle $G \to E \to B$. Any compact Lie group G is isomorphic to a subgroup of a special orthogonal group $SO(n)$. But $SO(n)$ acts freely (and preserves the orientation because $SO(n)$ is connected) on the highly connected Stiefel manifold $SO(n + N)/SO(N) = E$.

3. The obstruction to preserving the trace

We investigate the effect of adjoining an equivariant cell to a G–CW complex on the trace of the action. Let X be G–CW complex. The case to understand is the adjunction of an equivariant cell, because any finite G–CW complex is built in finitely many such steps. Recall that an equivariant n-cell $G/K \times e^n$, (where K is a closed subgroup, G acts on the first factor) is adjoined by extending a map $f : S^{n-1} \to X^K$ equivariantly to $\widetilde{f} : G/K \times S^{n-1} \to X$, the boundary of our n-cell. The next result establishes an obstruction to removing an equivariant cell and preserving the trace.

Key Lemma. *Let G be a compact Lie group, X a finite G–CW complex and $Y = (G/K \times e^n) \cup_{\widetilde{f}} X$ as above. Then there is an $\alpha \in H^{n+d}(B_K; \mathbb{Z})$ such that*

$$\mathrm{tr}(G,Y) \big| \, \mathrm{tr}(G,X) \big| \, \mathrm{tr}(G,Y) \ |\alpha|$$

where $d = \dim G/K$ and $|\alpha|$ is the (additive) order of α.

Proof: We know that $\mathrm{tr}(G,Y)\big|\,\mathrm{tr}(G,X)$ because there is a G-map from X to Y ([**G**], 6.2a). To show that $\mathrm{tr}(G,X)\big|\,\mathrm{tr}(G,Y)\ |\alpha|$, we first choose a closed oriented manifold E of connectivity $N > n+d$, where G acts freely on E. Then, as above, we have manifolds, $E/G = B$ and E/K. Now E/K is N-homotopy equivalent to B_K, so $H^{n+d}(B_K) \cong H^{n+d}(E/K)$. So the α we are seeking in $H^{n+d}(B_K)$, we may regard as in $H^{n+d}(E/K)$. Now E/K is a closed oriented manifold of dimension $\dim E/K = \dim E - \dim K = \dim B + \dim G - \dim K = \dim B + \dim G/K = m + d$, where m is the dimension of B. Now by Poincare duality $H^{n+d}(E/K) \cong H_{m-n}(E/K)$. Hence we can imagine the α in the theorem as an element in $H_{m-n}(E/K)$.

We construct α as follows. There is an element $\omega \in H_m((Y \times E)/G)$ so that $p_*(\omega) = \mathrm{tr}(G,Y)[B]$. Consider the Meyer-Vietoris exact sequence of the union $(Y \times E)/G = ((X \times E)/G) \cup ((G/K \times e^n \times E)/G)$, noting that $(G/K \times e^n \times E)/G = e^n \times E/K$ and also the intersection of the two subspaces is equal to $(G/K \times S^{n-1} \times E)/G$:

$$\to H_m((X \times E)/G) \oplus H_m(e^n \times (E/K)) \xrightarrow{i_* + j_*} H_m((Y \times E)/G) \xrightarrow{\delta}$$
$$H_{m-1}(S^{n-1} \times (E/K))| \xrightarrow{(k,\ell)_*} H_{m-1}((X \times E)/G) \oplus H_{m-1}(e^n \times (E/K))$$

Then $\delta(\omega) = [S^{n-1}] \times \alpha + 1 \times \beta \in H_{m-1}(S^{m-1} \times E/K)$ where α is some element in $H_{m-n}(E/K)$ and β is some element in $H_{m-1}(E/K)$. But note that $(k,\ell)_*(1 \times \beta) = 1 \times \beta \ \oplus \ g$ where g is some other term contained in the $H_{m-1}((X \times E)/G)$-summand. Hence, by the exactness of the Meyer-Vietoris exact sequence, β must be zero . Hence $\delta(\omega) = [S^{n-1}] \times \alpha$.

Now $|\alpha|\omega$ is in the kernel of δ, hence it is in the image of $i_* + j_*$ where i and j are the inclusions of $(X \times E)/G$ and $(e^n \times (E/K))$ into $(Y \times E)/G$ respectively. So there are u and v so that $i_*(u) + j_*(v) = |\alpha|\omega$. So $p_*(i_*(u) + j_*(v)) = p_*(|\alpha|\omega)$. We have $|\alpha| \operatorname{tr}(G,Y)[B] = p_*(|\alpha|\omega) = p_*(i_*(u) + j_*(v)) = a[B] + b[B] = (a+b)[B]$ for some integers a and b. Now $\operatorname{tr}(G,X)$ divides a and $\operatorname{tr}(G,G/K)$ divides b, so $(a+b)$ is a multiple of the greatest common divisor of $\operatorname{tr}(G,X)$ and $\operatorname{tr}(G,G/K)$. But G/K maps equivariantly into X, so $\operatorname{tr}(G,X)$ divides $\operatorname{tr}(G,G/K)$. Hence $\operatorname{tr}(G,X)$ divides $a+b$ and hence divides $|\alpha|\ \operatorname{tr}(G,Y)$ as was to be shown. □

In order to apply the key lemma we need some control over the cohomology of the isotropy subgroup K. The easiest case is when K is trivial, so all cohomology in positive dimensions vanish. We call two G spaces X and Y *G–related* if there are G-maps $f : X \to Y$ and $f' : Y \to X$. The *singular set* of a G-space X, denoted X^S, consists of all points in X with nontrivial isotropy subgroups.

THEOREM 3. *Let G be a compact Lie group and let X be a G- space which is G-related to a finite G–CW complex. Then* $\operatorname{tr}(G,X) = \operatorname{tr}(G,X^S)$, *when X^S is not empty.*

Proof: If X is G-related to Y then the singular sets X^S and Y^S are also G-related. Hence $\operatorname{tr}(G,X) = \operatorname{tr}(G,Y)$ and $\operatorname{tr}(G,X^S) = \operatorname{tr}(G,Y^S)$, so we only need to prove $\operatorname{tr}(G,Y) = \operatorname{tr}(G,Y^S)$. The finite G–CW complex Y is built from the subcomplex Y^S by adding equivariant cells with trivial isotropy subgroup. But the trace does not change when we add free cells, by the key lemma. □

4. Trace and orbit size

When G is a finite group, a naive invariant measuring the nontriviality of the G-action on X is the *orbit size*, denoted $\operatorname{os}(G,X)$. This is defined as the greatest common divisor of the cardinalities of all the orbits. That is

$$\operatorname{os}(G,X) = \gcd\{[G : G_x] : x \in X\}.$$

In this section we compare the properties of the trace and the orbit size.

(i) $\operatorname{tr}(G,X) \big| \operatorname{os}(G,X)$ if X is an arbitrary space ([**G**], 6.2a and 6.7b).

(ii) Suppose that G_p is a Sylow p–subgroup of G. Then $\operatorname{tr}(G,X) = \Pi \operatorname{tr}(G_p,X)$ where the product is over all primes p dividing the order of G. ([**G–O**], theorem 14; [**O**], theorem 2.5)

(iii) $\operatorname{os}(G,X) = \Pi \operatorname{os}(G_p,X)$ where the product is taken over all primes p dividing the order of G.

(iv) $\operatorname{tr}(G,X) = \operatorname{os}(G,X)$. If all Sylow p–subgroups of G are elementary abelian and X is a finite complex. This follows from (ii) and (iii) and ([**G**], theorem 7.4) or ([**B2**], theorem 1.1).

(v) $\operatorname{os}(G,X) \big| \Lambda_f$. where X is a finite complex and $f : X \to X$ is a G–map and the Lefschetz number of f is denoted by Λ_f. ([**O**],appendix)

Now we show that the trace equals the orbit size when X is low dimensional. On the other hand for cyclic groups acting on closed oriented manifolds, we can find examples where the trace and orbit size are any pair of positive integers subject to condition (i). These results will follow from the key lemma and theorem 3.

COROLLARY 4. *Let G be a finite group and let X be G-related to a finite, one dimensional G–CW complex (a G-graph). Then* $\mathrm{tr}(G,X)=\mathrm{os}(G,X)$.

Proof: Assume X^S is not empty. Since $\mathrm{tr}(G,X)$ and $\mathrm{os}(G,X)$ are invariants of G-relatedness, it suffices to prove this for a finite, one dimensional G–CW complex X. For any finite (isotropy) group K, we know $H^1(K;\mathbb{Z})$ is trivial, hence by the key lemma the trace of the zero-skeleton is equal to $\mathrm{tr}(G,X)$. But for a finite discrete set X°, $\mathrm{tr}(G,X^\circ)=\mathrm{os}(G,X^\circ)$. When the singular set is empty, i.e., when the action is free then $\mathrm{tr}(G,X)=|G|=\mathrm{os}(G,X)$ (G a finite group and X a finite G–CW complex). The first equality follows from the key lemma by peeling off free equivariant cells till we get to the zero skeleton. □

COROLLARY 5. *Let G be a finite group acting smoothly on a connected manifold M. Assume that there is a free orbit and either: a) M is a compact surface; or b) M is an oriented 3-manifold with an orientation preserving action. Then* $\mathrm{tr}(G,M)=\mathrm{os}(G,M)$.

Proof: M is a finite G–CW complex [**I2**]. Since there is a free orbit the hypothesis implies that the singular set M^S is at most one dimensional, (because the action is orientation preserving in the three manifold case the codimension of the singular set must be greater than 1). Therefore Theorem 3 and Corollary 4 imply $\mathrm{tr}(G,M)=\mathrm{os}(G,M)$. □

LEMMA 6. *Let $\phi:\mathbb{Z}_{p^{n+1}}\to\mathbb{Z}_{p^n}$ be the cononical epimorphism. Thinking of $\mathbb{Z}_{p^n}$ as a subgroup of S^1, we have the standard action on S^{2k+1} given by complex multiplication on the unit sphere in complex $(k+1)$–space. Let $\mathbb{Z}_{p^{n+1}}$ act on S^{2k+1} via ϕ. Then* $\mathrm{os}(\mathbb{Z}_{p^{n+1}},S^{2k+1})=p^n$ *and* $\mathrm{tr}(\mathbb{Z}_{p^{n+1}},S^{2k+1})=\max(p^{n-k},1)$.

Proof: Since the action is orientation preserving, the trace equals the fiber number ([**G**], 6.4) The fiber number is given by the exponent of the transgression of the fundamental class $[\overline{S}^{2k+1}]$ in the cohomology Serre spectral sequence of the Borel construction:

$$H^{2k+1}(S^{2k+1})\cong E_2^{0,2k+1}\to E_2^{2k+2,0}\cong H^{2k+2}(\mathbb{Z}_{p^{n+1}})$$

(because there are only 2 nonzero rows at the E^2 level). The action being induced from the action of $\mathbb{Z}_{p^n}$, the transgression map factors through $H^{2k+2}(\mathbb{Z}_{p^n})\to H^{2k+2}(\mathbb{Z}_{p^{n+1}})$, by the naturality of the Serre spectral sequence. $H^*(\mathbb{Z}_{p^n})$ is generated by the powers of α in $H^2(\mathbb{Z}_{p^n})\cong\mathbb{Z}_{p^n}$, and $\phi^*(\alpha)=p\beta$ where β is an analogous generator for $H^2(\mathbb{Z}_{p^{n+1}})$. So $\phi^*(\alpha^{k+1})=p^{k+1}\beta^{k+1}$. The image of $[\overline{S}^{2k+1}]$ under the transgression in the Borel construction of the action of $\mathbb{Z}_{p^n}$ is a generator of $H^{2k+2}(\mathbb{Z}_{p^n})$ because $\mathbb{Z}_{p^n}$ is acting freely. Thus the exponent of the image of $[\overline{S}^{2k+1}]$ in $H^{2k+2}(\mathbb{Z}_{p^{n+1}})\cong\mathbb{Z}_{p^{n+1}}=<\beta^{k+1}>$ is $p^{n+1}/p^{k+1}=p^{n-k}$ if $k\le n$ and 1 otherwise. Also $\mathrm{os}(\mathbb{Z}_{p^{n+1}},S^{2k+1})=p^n$ because every orbit has $p^{n+1}/p=p^n$ elements. □

Let G and H act on spaces X and Y respectively. We say that $G \times H$ acts on $X \times Y$ with the *product action* if $g \times h(x \times y) = g(x) \times h(x)$ for all $g \in G$, $h \in H$, $x \in X$, $y \in Y$.

LEMMA 7.. *If $(G \times H)$ is a product action, and if the order of G is relatively prime to the order of H, then*
a) $\mathrm{tr}(G \times H, X \times Y) = \mathrm{tr}(G, X) \cdot \mathrm{tr}(H, Y)$
b) $\mathrm{os}(G \times H, X \times Y) = \mathrm{os}(G, X) \cdot \mathrm{os}(H, Y)$

Proof: We identify G with the subgroup $G \times 1 \subset G \times H$ and H with the subgroup $1 \times H$. Then the composition $X \to X \times Y \to X$ given by $x \mapsto (x \times y_0) \mapsto x$ is a composition of G–maps. Thus X and $X \times Y$ are G–related. Both trace and orbit size are preserved under G–relatedness, hence $\mathrm{tr}(G, X) = \mathrm{tr}(G, X \times Y)$ and $\mathrm{os}(G, X) = \mathrm{os}(G, X \times Y)$. Similarly $\mathrm{tr}(H, Y) = \mathrm{tr}(H, X \times Y)$ and $\mathrm{os}(H, Y) = \mathrm{os}(H, X \times Y)$. Now it follows from (ii) and the fact that $|G|$ and $|H|$ are relatively prime that $\mathrm{tr}(G \times H, X \times Y) = \mathrm{tr}(G, X \times Y) \cdot \mathrm{tr}(H, X \times Y)$ which equals $\mathrm{tr}(G, X) \cdot \mathrm{tr}(H, Y)$ as required. Similarly, using (iii) and the relative primeness of $|G|$ and $|H|$, we get $\mathrm{os}(G \times H, X \times Y) = \mathrm{os}(G, X) \cdot \mathrm{os}(H, Y)$.

THEOREM 8. *Let m and n be positive integers with m dividing n. There exists an action (G, M) where G is cyclic, M is a closed oriented manifold and the action is orientation preserving and effective, such that* $\mathrm{tr}(G, M) = m$ *and* $\mathrm{os}(G, M) = n$.

Proof: We factor m and n into prime factors. Then $m = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$ and $n = p_1^{e_1+f_1} p_2^{e_2+f_2} \dots p_k^{e_k+f_k}$, where $0 \le e_i, f_i$ for all $i = 1, \dots, k$. For each i consider the action $(\mathbb{Z}_{p_i}^{e_i+f_i+1}, S^{2f_i+1})$ as described in Lemma 6. So the trace of the action is $p_i^{e_i}$ and the orbit size is ${p_i}^{e_i+f_i}$. Now let G be the product of the $\mathbb{Z}_{p_i}^{e_i+f_i+1}$, so G is a cyclic group. Let X be the product of the S^{2f_i+1}. Then by lemma 7, the product action (G, X) has trace equal to m and orbit size equal to n . This proves the theorem except for the conclusion that the action should be effective. We attach a fee G–cell to X to get the action (G, Y) which is effective. By the key lemma, $\mathrm{tr}(G, Y)$ equals m; and $\mathrm{os}(G, X)$ obviously equals n. But now Y is not a manifold. Using lemma 2 we can find a compact oriented manifold on which G acts effectively, preserving orientation, and which retracts equivariantly onto Y . Then doubling the manifold and the action we get a compact oriented manifold M on which G acts effectively, preserving orientation, and which retracts equivariantly onto Y. Thus the trace and orbit size of (G, M) still equal m and n . □

Finally we show that corollary 4 is true only for dimension of X equal to one, and corollary 5 needs the hypothesis that there is a free orbit, by considering the following example which also gives a non-trivial use of the key lemma.

EXAMPLE 9. *Let $\mathbb{Z}_4$ act on S^2 through the epimorphism $\mathbb{Z}_4 \to \mathbb{Z}_2$ and then via the antipodal action. Then* $\mathrm{tr}(\mathbb{Z}_4, S^2) = 1$, *and* $\mathrm{os}(\mathbb{Z}_4, S^2) = 2$.

Proof: By lemma 6, $\mathrm{tr}(\mathbb{Z}_4, S^3) = 1$. But S^3 is obtained from S^2 by adding an equivariant 3 cell of isotropy type $\mathbb{Z}_2$. Since $H^3(\mathbb{Z}_2) = 0$ we get $\mathrm{tr}(\mathbb{Z}_4, S^2) = \mathrm{tr}(\mathbb{Z}_4, S^3) = 1$ by the key lemma. □

5. The trace and the exponent

We have already mentioned that the trace is closely related to the exponent of ([**A**], definition 1.4 and 3.1) . The trace is defined for any action while the exponent is defined only for actions of finite groups acting on connected finite dimensional CW–complexes. We will show that the trace equals the exponent, but only for finite complexes. Whether they are equal for actions on non-compact, finite dimensional complexes is open. We will suggest an extension of the definition of the exponent to the case of compact Lie groups acting on possibly disconnected G–CW complexes.

PROPOSITION 10. *If G is a finite group and X is a connected compact G–CW complex, then $\operatorname{tr}(G,X) = e_G(X)$, i.e. the trace equals the exponent.*

Proof: If X is a connected orientable manifold and the finite group G acts preserving orientation, then the trace and the exponent both equal the fibre number ([**G**], theorem 6.4; [**A**], corollary 3.13). Now lemma 2 states that a compact G–CW complex is an equivariant retract of of a closed oriented G–manifold. Since both the trace and the exponent are invariant under equivariant retracts, they are equal. □

It would be useful to know if the trace equaled the exponent for non-compact finite dimensional G–CW complexes since the exponent works just as well in that case as in the compact case. For example $e_G(X) = e_G(X^S)$. So if trace equals exponent, then our main theorem, theorem 3, could be extended to non-compact situations for finite G.

We propose to define $e_G(X)$ in the case where X is a possibly disconnected finite dimensional G–CW complex and G is a connected Lie group. First if G is finite and acts transitively on the set of connected components, where $e_G(X)$ is the subgroup of G which fixes a component G_0, we define

$$e_G(X) = [G : G_0] \cdot e_{G_0}(X)$$

Then if X is generally disconnected, we note that X is the union of spaces X_i on which G acts transitively and we define

$$e_G(X) = \gcd\{e_G(X_i)\}$$

Finally if G is a compact Lie group we propose the definition

$$e_G(X) = \operatorname{lcm}\{e_H(X)\} \text{ for all finite subgroups } H \text{ of } G.$$

The new exponent $e_G(X)$ satisfies all the relevant properties listed in the introduction of [**A**] when G is finite. Also, for G finite and X compact, the trace equals the exponent. This follows because the trace satisfies the first two equations above ([**O**], proposition 3.3;

[**G**],p. 397). In the case of G compact and X compact we no longer know if they are equal, but $e_G(X)$ divides $\mathrm{tr}(G, X)$ since $\mathrm{tr}(H, X)$ divides $\mathrm{tr}(G, X)$ by ([**G**], 6.2b). (It is unusual for a useful invariant to divide the trace, it is usually the other way.) The trace would be equal to the exponent if the answer to the following question is yes. Does

$$\mathrm{tr}(G, X) = \mathrm{lcm}\ \{\mathrm{tr}(H, X)\} \text{ for all finite subgroups } H \text{ of } G\ ?$$

A similar question, whose affirmative answer would imply that trace equals exponents when G is finite and X is finite dimensional, is the following. Does

$$\mathrm{tr}(G, X) = \mathrm{gcd}\ \{\mathrm{tr}(G, Y)\} \text{ for all finite } G\text{-subcomplexes } Y \text{ of } X\ ?$$

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