Gottlieb

Rules of Algebra

Let $a,\ b,\ c$ be any real numbers

  1) $a + b = b + a$

  2) $a + (b + c) = (a + b) + c$

  3) $a + (-1)a = 0$ (also written $a - a = 0$)

  4) $a + 0 = a$

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  5) $ab = ba$

  6) $a(bc) = (ab)c$

  7) $aa^{-1} = 1$ (also written $\dfrac{a}{a} = 1$) [if $a \neq 0$]

  8) $1 \cdot a = a,\ 0 \cdot a = 0,\ (-a)(-b) = ab$

  9) $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd} = (ad + bc)(bd)^{-1}$

10) $a(b + c) = ab + ac$

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11) $a^n = a \cdot a \ldots a$ (n–times) where n is a positive integer

12) $a^0 = 1$

13) $a^{-n} := \frac{1}{a^n}$ [$a \neq 0$]

14) $a^{n/m} := \sqrt[m/n]{a}$ (i.e. $(a^{n/m})^m = a^n$) [$a \geq 0$ if m is even]

15) $a^{r+s} = a^r a^s$ [for r, s any numbers and $a > 0$]

16) $(a^r)^s = a^{rs}$ [for r, s any numbers and $a > 0$]

17) $(ab)^r = a^r b^r$ [for r any number and $a > 0,\ b > 0$]

18) $1^r = 1$

19) $0^r = 0$ [if $r > 0$]

Note that

20) $a - b := a + (-1)b$

21) $\dfrac{a}{b} := ab^{-1}$

22) $a^{m/n} := \sqrt[m/n]{a}$ where $(a^{m/n})^n = a^m$

So $\sqrt{}$, subtraction, and $\dfrac{a}{a}$ are not necessary.