Hyperspectral SAR

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Distribution A

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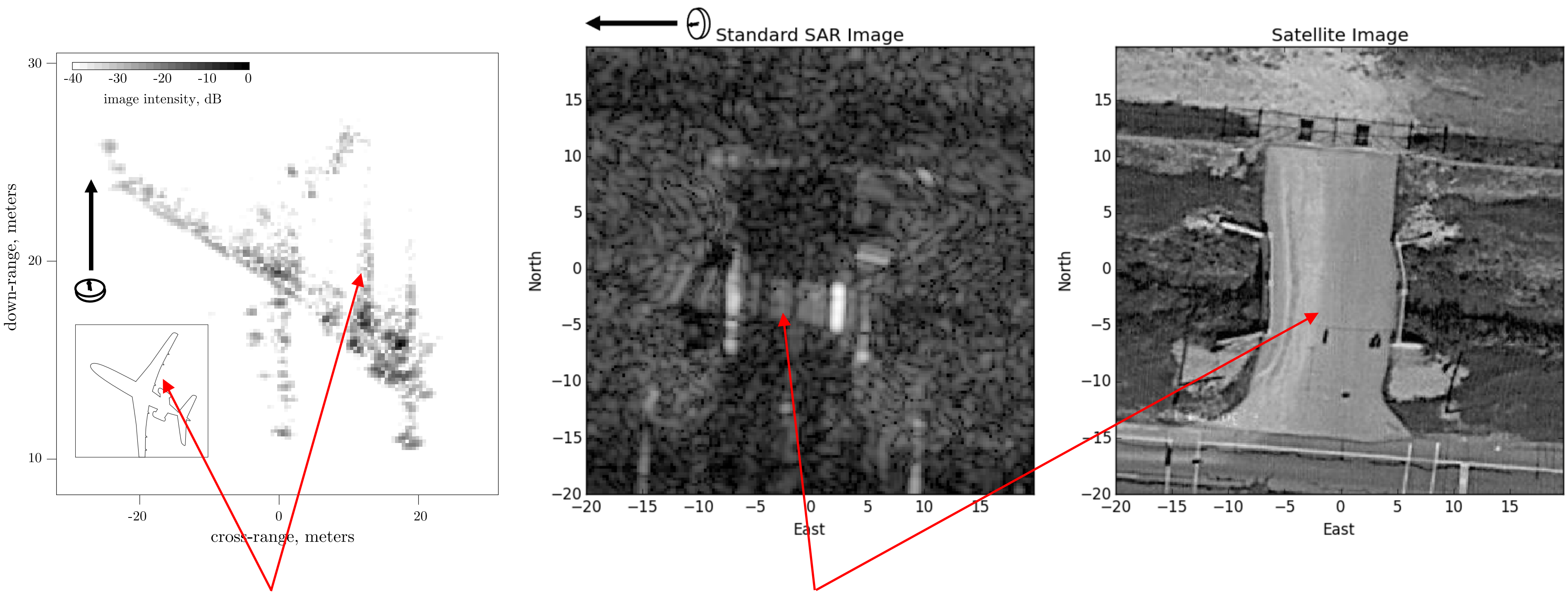
Outline

- Problem definition & related work
- Physical model & imaging algorithm
- Point spread function (PSF)-based resolution analysis
- Circular SAR example using Xpatch data
- Conclusions

Problem

- Typical SAR imaging techniques neglect dispersion in scene reflectivity
- Below are examples of inadequately modeled structural dispersion



streaks above the engine inlets

streak across bridge span

Previous work

MATRIX RESEARCH

- Waveform and geometry specific
  - Chirps → de-chirped phase history data
    - Zak-space estimation (Tolimieri). Used start-stop approximation.
  - *We wanted a method which was not restricted to a specific waveform or flight geometry*
- Parametric dispersion models
  - Extract simple parametric models from distortions in standard SAR (Gilman & Tsynkov) and ISAR (Borden) images
  - *We wanted to avoid restrictions to parametric models*
- Frequency-domain-based processing
  - Sub-banding (Albanese & Medina), frequency-domain filters (e.g., Garren 2002)
    - Frequency-domain pixel representations may have non-zero negative-time coefficients
  - *These methods to not explicitly enforce a causal model for dispersion*

Model

- Scalar wave propagation from antenna to scene

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{E}(t,\boldsymbol{x}) = s(t,\boldsymbol{x})$$

- Time-dependent reflectivity model

$$\mathcal{E}^{tot} = \mathcal{E}^{in} + \mathcal{E}^{sc}$$

Models scattering object

$$s = s^{in} + s^{sc}$$

Models transmit antenna

$$s^{sc}(t,\boldsymbol{x}) = -\int_{t-\Delta T}^{t} v(t-t',\boldsymbol{x})\mathcal{E}^{tot}(t',\boldsymbol{x})dt'$$

supported on $0 \le t - t' \le \Delta T$

- "Standard" SAR uses $v(t-t',\boldsymbol{x}) = v(\boldsymbol{x})\delta(t-t')$

Incident Field Model

$$s^{in}(t, \boldsymbol{x} - \boldsymbol{\gamma}(t)) = -j(t, \boldsymbol{x} - \boldsymbol{\gamma}(t))$$

$$\mathcal{E}^{in}(t^{inc}, \boldsymbol{x}) = \int g(t^{inc} - t', \boldsymbol{x} - \boldsymbol{y}) j(t', \boldsymbol{y} - \boldsymbol{\gamma}(t')) dt' d\boldsymbol{y}$$

$$\mathcal{E}^{in}(t^{inc}, \boldsymbol{x}) \approx \int \overbrace{j(t^{tr}_{t^{inc},\boldsymbol{x}}, \boldsymbol{z})}^{\text{Current density on antenna}} \left[4\pi|\boldsymbol{x} - \boldsymbol{\gamma}(t^{tr}_{t^{inc},\boldsymbol{x}})| \overbrace{\left[1 + \left(\widehat{\boldsymbol{\gamma}(t^{tr}_{t^{inc},\boldsymbol{x}}) - \boldsymbol{x}}\right) \cdot \dot{\boldsymbol{\gamma}}(t^{tr}_{t^{inc},\boldsymbol{x}})/c\right]}^{\text{Doppler scale factor}}\right]^{-1} d\boldsymbol{z}$$

$$\approx \frac{f\left(t^{inc} - R_{\boldsymbol{x}}(t^{tr}_{t^{inc},\boldsymbol{x}})/c\right) \tilde{W}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{x}}(t^{tr}_{t^{inc},\boldsymbol{x}})\right)}{4\pi R_{\boldsymbol{x}}(t^{tr}_{t^{inc},\boldsymbol{x}})}$$

Transmitted waveform

One-way range at transmit time

Antenna beam pattern (transmit)

Integral over antenna aperture

Transmit time associated with one-way propagation

Resulting scattered field

- Lippmann-Schwinger equation

$$\mathcal{E}^{\rm sc}(t,\boldsymbol{x}) = -\int\int g(t-\tau,\boldsymbol{x}-\boldsymbol{z})s^{sc}(\tau,\boldsymbol{z})d\tau d\boldsymbol{z}$$
$$= \int\int g(t-\tau,\boldsymbol{x}-\boldsymbol{z})\int\Big(v(\tau-t',\boldsymbol{z})\mathcal{E}^{tot}(t',\boldsymbol{z})\Big)dt'd\tau d\boldsymbol{z}$$

where $g(t,\boldsymbol{x}) = \frac{\delta(t-|\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|} = \int \frac{e^{i\omega(t-|\boldsymbol{x}|/c)}}{8\pi^2|\boldsymbol{x}|}d\omega$

- Born-approximated Lippmann-Schwinger equation

replace $\mathcal{E}^{tot}$ with $\mathcal{E}^{in}$

$$\mathcal{E}^{sc}_B(t,\boldsymbol{x}) = \iint g(t-t^{rad},\boldsymbol{x}-\boldsymbol{y})\underbrace{\int_0^{\Delta T} v(\Delta\tau,\boldsymbol{y})\mathcal{E}^{in}(t^{rad}-\Delta\tau,\boldsymbol{y})d\Delta\tau}_{s^{sc}_B(t^{rad},\boldsymbol{y})}\,dt^{rad}d\boldsymbol{y}$$

$$= \int \frac{\delta(t-t^{rad}-|\boldsymbol{x}-\boldsymbol{y}|/c)}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}s^{sc}_B(t^{rad},\boldsymbol{y})dt^{rad}d\boldsymbol{y}$$

Imaging Algorithm

- Weighted matched filter

Cutoff function

$$I(\Delta\tau, \boldsymbol{z}) = \int \frac{R_{\boldsymbol{z}}(t^{rec}) R_{\boldsymbol{z}}(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}}) f^*(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}}) \chi_{\Delta\tau,\boldsymbol{z}}(t^{rec})}{\tilde{A}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{z}}(t^{rec})\right) \tilde{W}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{z}}(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}})\right)} d(t^{rec}) dt^{rec}$$

Measured data

Receive beam pattern

Transmit beam pattern

- Round-trip travel time $t^{rec} - t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}} = \dfrac{R_{\boldsymbol{z}}(t^{rec})}{c} + \Delta\tau + \dfrac{R_{\boldsymbol{z}}(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}})}{c}$

- Resulting point spread function (PSF)

$$I(\Delta\tau, \boldsymbol{z}) = \int_{\Omega_{\boldsymbol{z}}} \int_0^{\Delta T} K(\Delta\tau, \boldsymbol{z}, \Delta\sigma, \boldsymbol{y}) v(\Delta\sigma, \boldsymbol{y}) d\Delta\sigma d\boldsymbol{y}$$

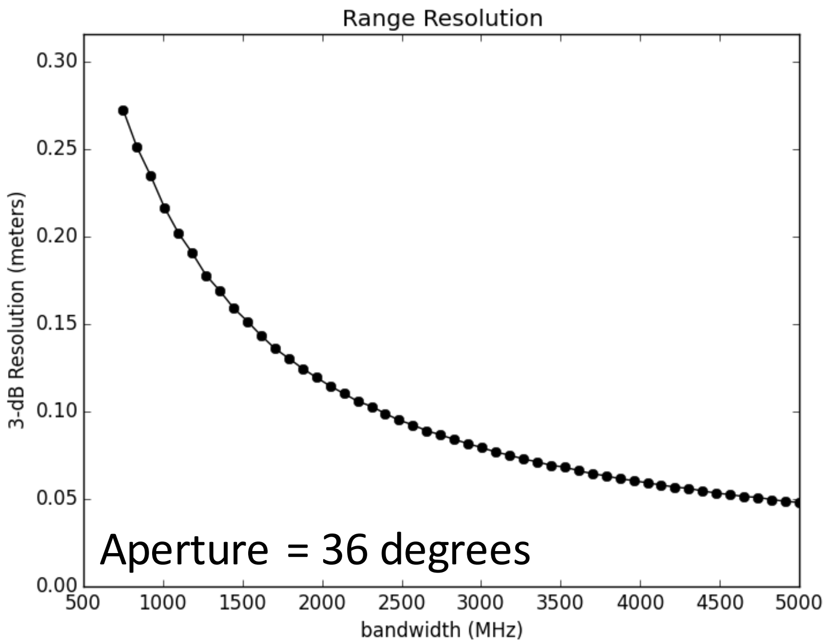
$$K(\Delta\tau, \boldsymbol{z}, \Delta\sigma, \boldsymbol{y}) = \int_{\Delta T^{rec}_{\Delta\tau,\boldsymbol{z}}} \left(\frac{R_{\boldsymbol{z}}(t^{rec}) R_{\boldsymbol{z}}(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}})}{R_{\boldsymbol{y}}(t^{rec}) R_{\boldsymbol{y}}(t^{tr}_{t^{rec},\Delta\sigma,\boldsymbol{y}})} \right.$$

$$\left. \frac{\tilde{W}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{y}}(t^{tr}_{t^{rec},\Delta\sigma,\boldsymbol{y}})\right) \tilde{A}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{y}}(t^{rec})\right)}{\tilde{W}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{z}}(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}})\right) \tilde{A}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{z}}(t^{rec})\right)} \right) f^*(t^{tr}_{t^{rec},\Delta\tau,\boldsymbol{z}}) f(t^{tr}_{t^{rec},\Delta\sigma,\boldsymbol{y}}) dt^{rec}$$

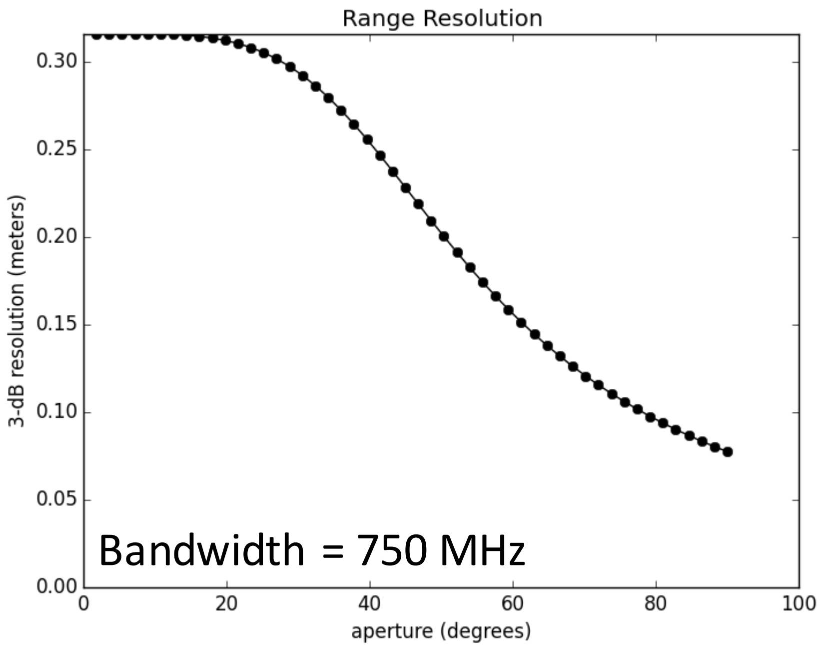
Range Resolution: 3-dB Main Lobe

- Circular SAR example, 40-degree elevation
- Chirp time-bandwidth product = 100

Fixed Aperture + Increasing Bandwidth



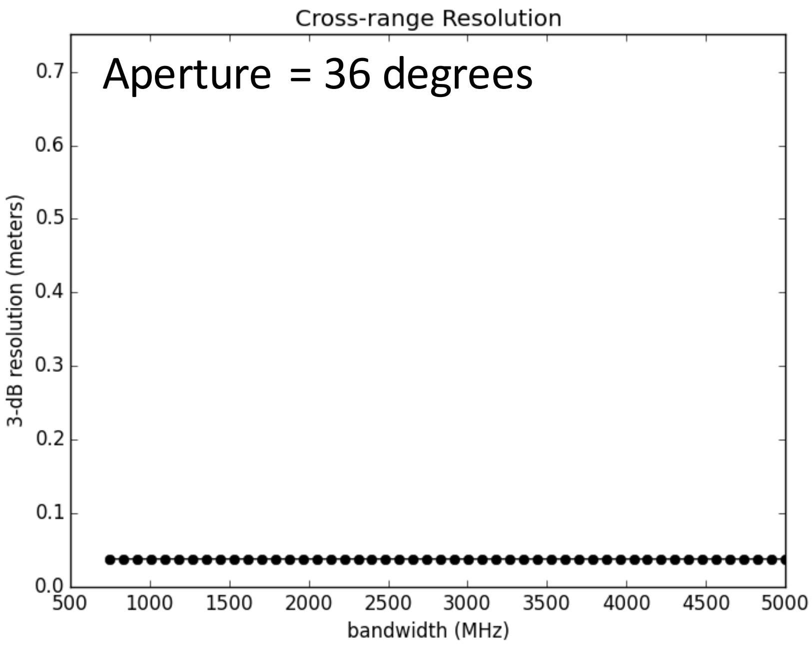
Increasing Aperture + Fixed Bandwidth



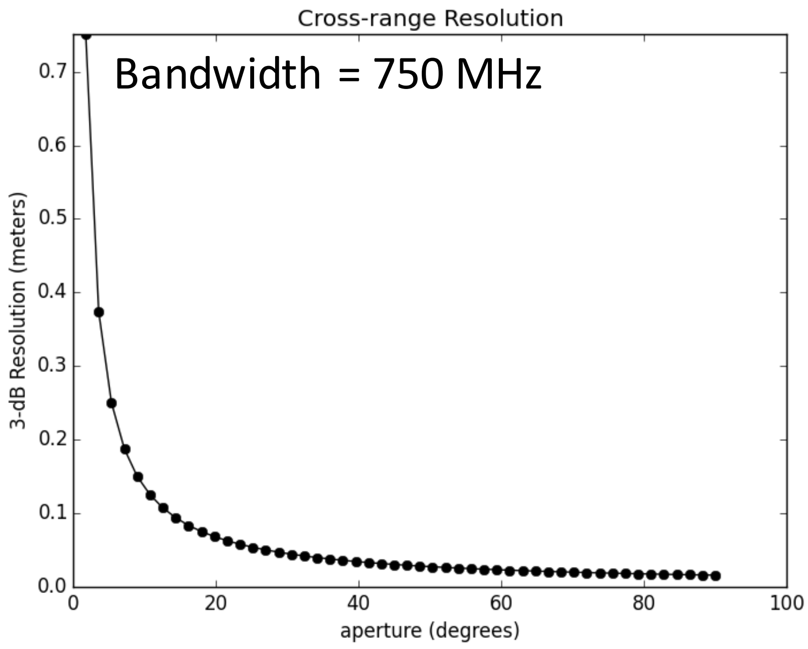
Cross-Range Resolution: 3-dB Main Lobe

- Circular SAR example, 40-degree elevation
- Chirp time-bandwidth product = 100

Fixed Aperture + Increasing Bandwidth



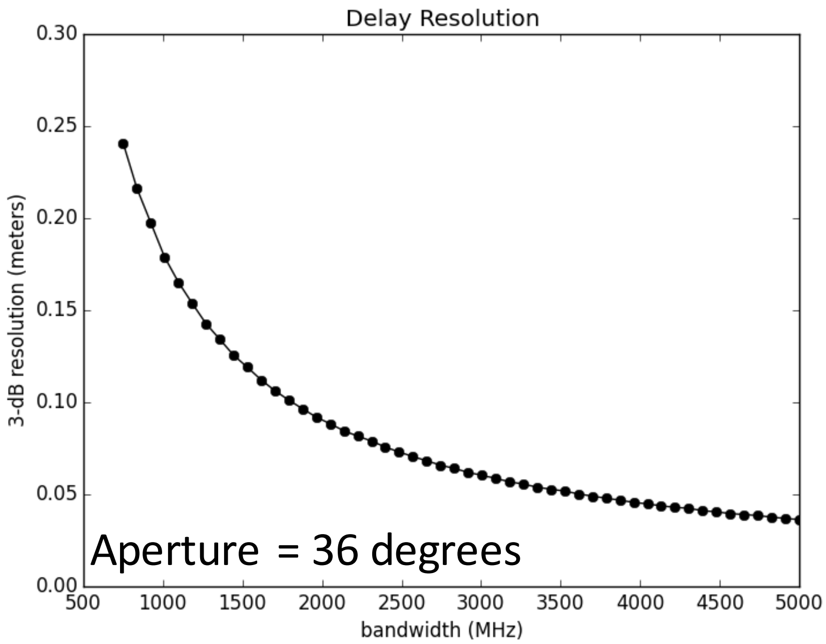
Increasing Aperture + Fixed Bandwidth



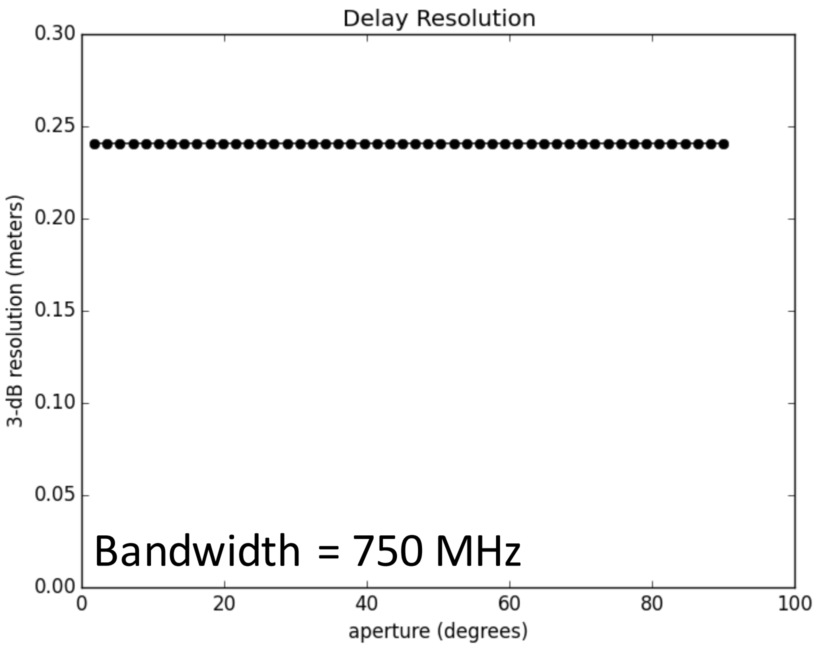
Delay Resolution: 3-dB Main Lobe

- Circular SAR example, 40-degree elevation
- Chirp time-bandwidth product = 100

Fixed Aperture + Increasing Bandwidth



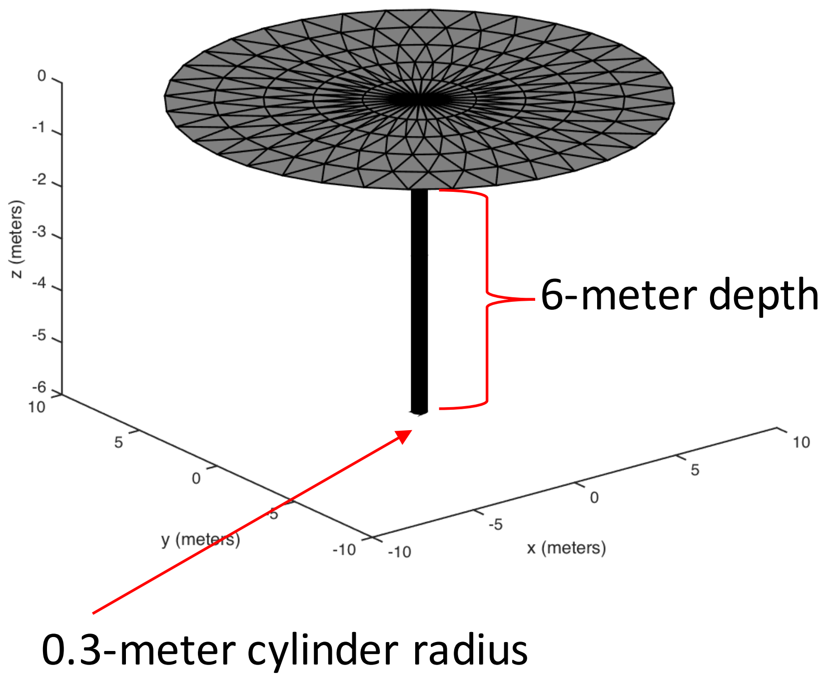
Increasing Aperture + Fixed Bandwidth



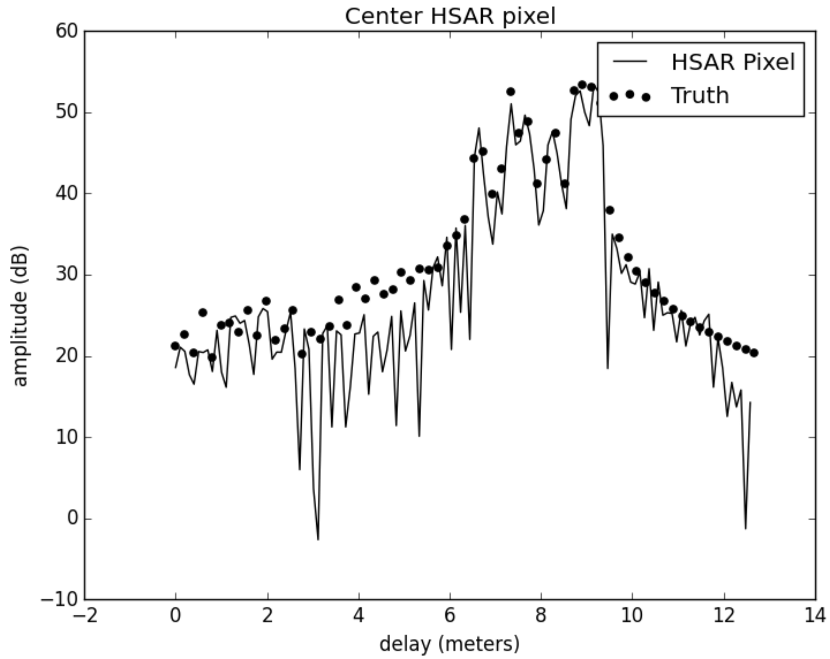
CSAR example: “bottom hat”

- Circular SAR with 36-degree aperture, 40-degree elevation angle
- 190 km flight-path radius, 69 m/s velocity
- Tukey-windowed 745 MHz chirp. T*BW = 100. Center frequency = 10 GHz

Closed-end Cylinder Target



Truth Vs. Reconstructed Reflectivity



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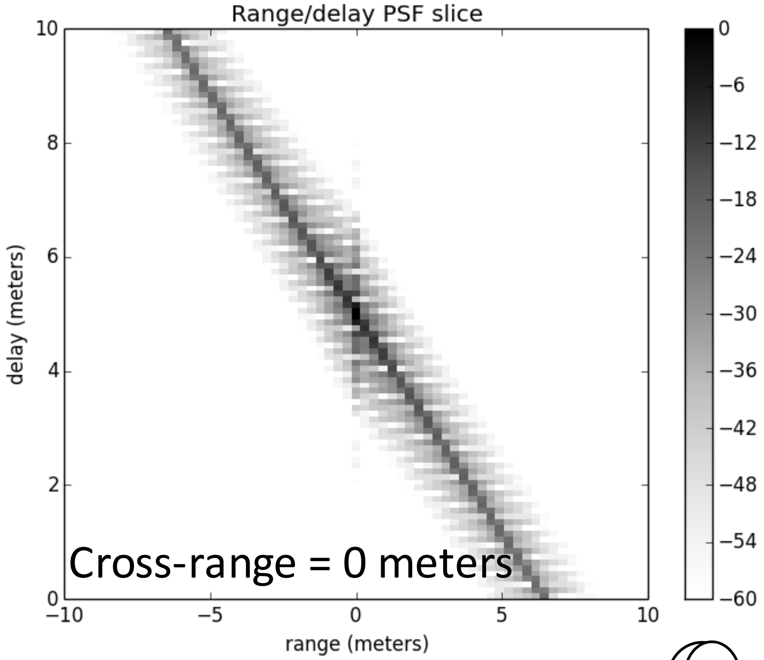
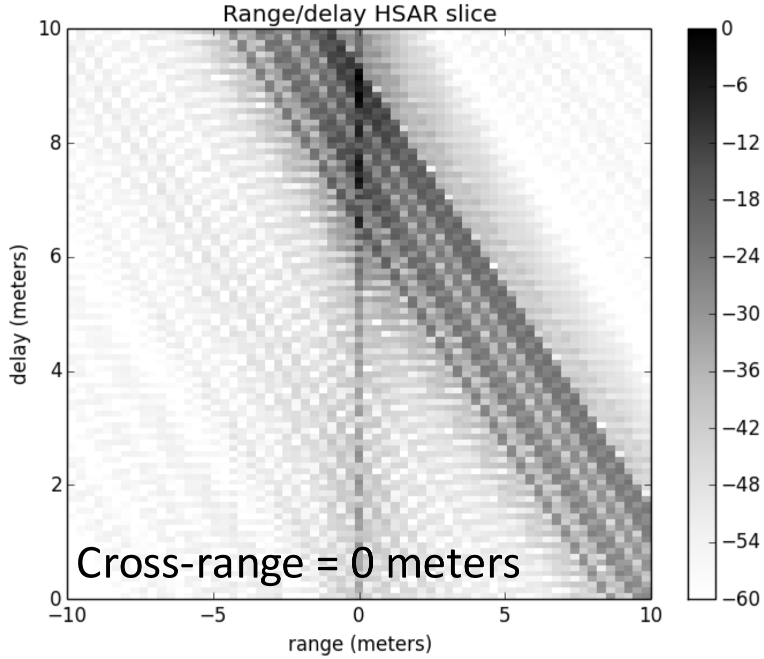
Point Spread Function
Delay Vs. Range



HSAR Image
Delay Vs. Range



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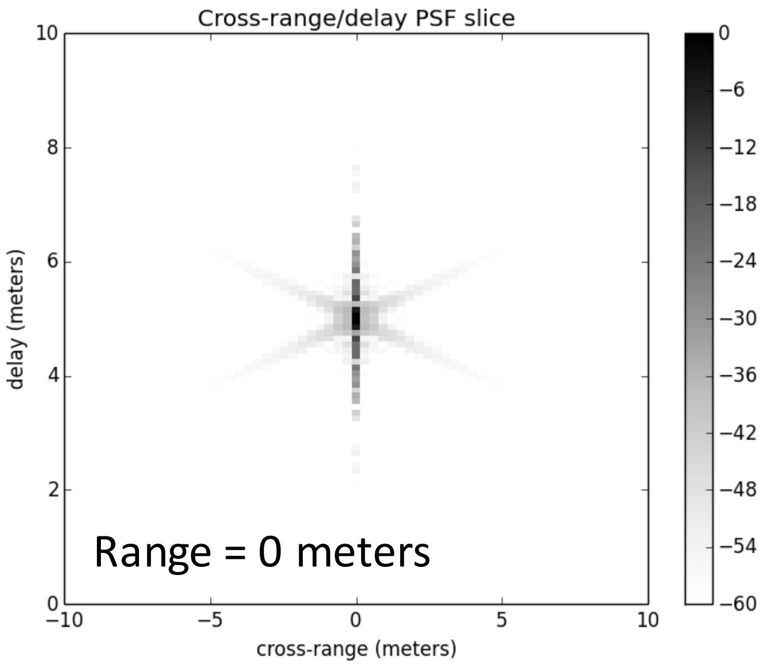
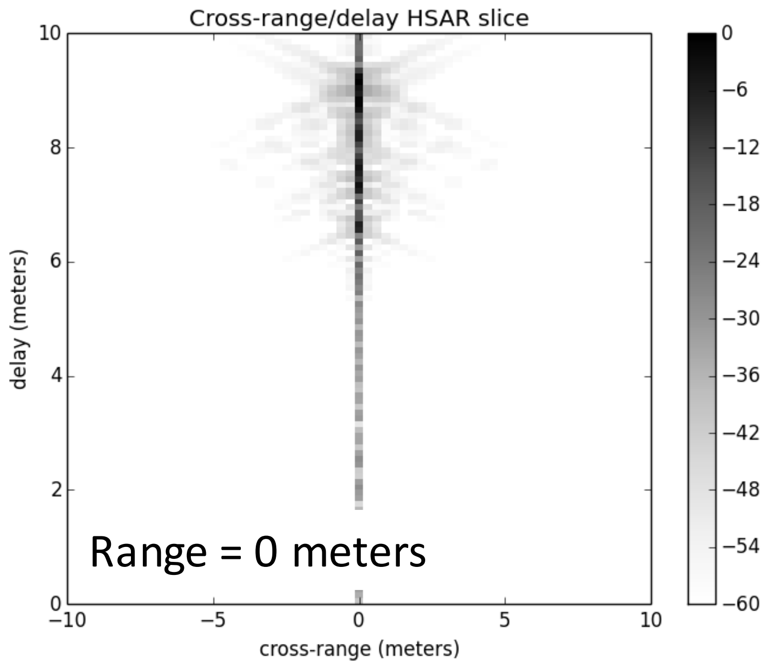
Point Spread Function
Delay Vs. Cross Range



HSAR Image
Delay Vs. Cross Range



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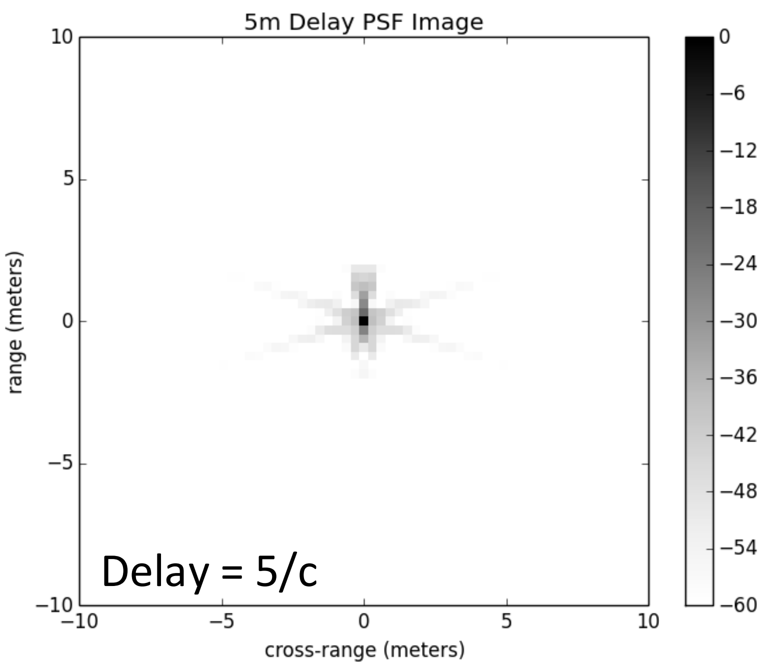
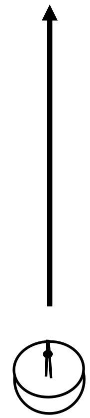
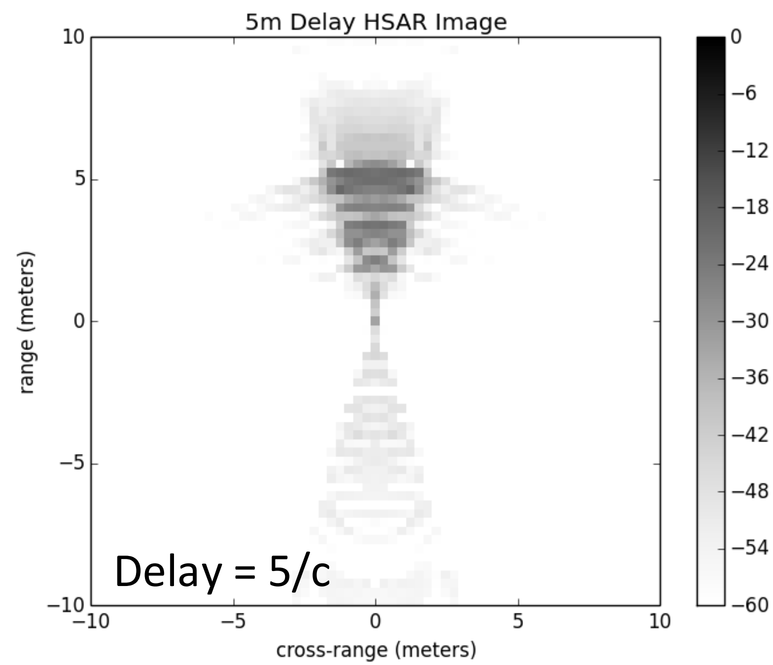
Point Spread Function
Range Vs. Cross Range





HSAR Image
Range Vs. Cross Range



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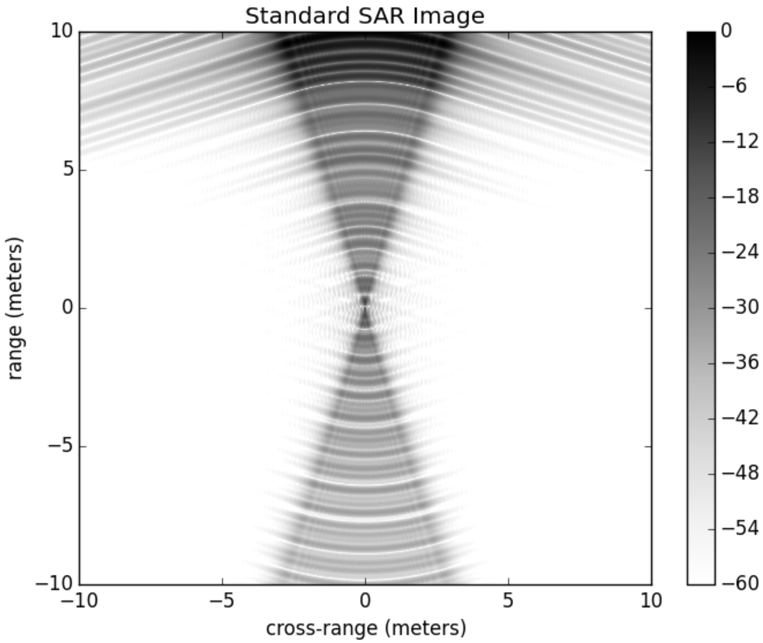
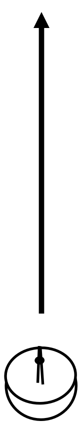
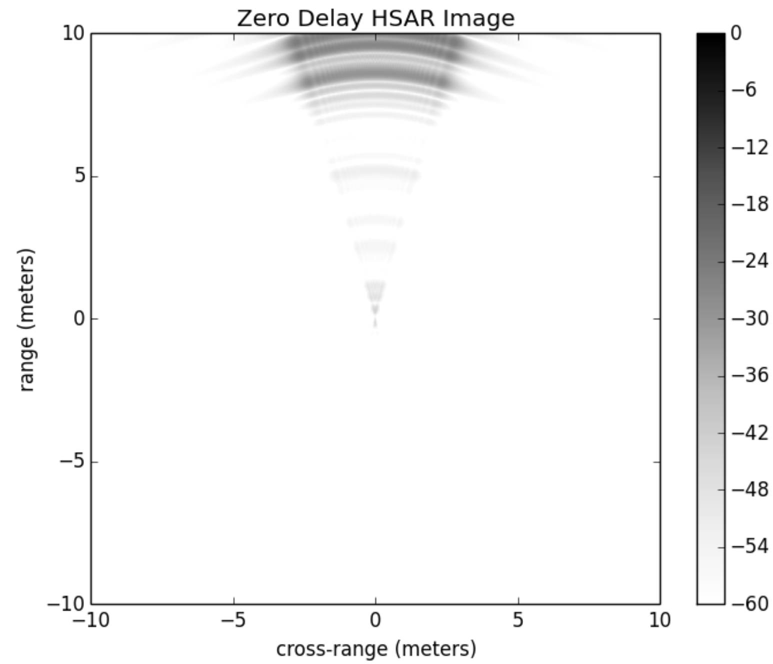
Start-stop + Matched Filter + NUFFT





HSAR Image, $\Delta\tau = 0$



Conclusions

- We have developed and analyzed a time-domain algorithm for HSAR which
  - accommodates *arbitrary transmit strategies*
  - naturally restricts scene-reflectivity estimates to *causal* functions

- Point spread function (PSF)-based resolution analysis indicates
  - When BW and aperture are varied independently we see the following enhancements in resolution:

	BW	Aperture
Range	✔	✔
Cross Range	✘	✔
Delay	✔	✘

- Based on our experiments with Xpatch-generated data, it is possible to reconstruct structural dispersion parameters provided *sufficient bandwidth* and *aperture* (as predicted by the PSF)

Ferrara, Homan, and Cheney, "Hyperspectral SAR," submitted to IEEE Transactions on Geoscience and Remote Sensing