WORKSHOP ON INVERSE PROBLEMS IN SCATTERING AND IMAGING

Saturday, April 23, 2016

Purdue University, West Lafayette, Indiana

Organizers: Peijun Li, Jie Shen (Chair), Plamen Stefanov

Check-In and Refreshments Beering Hall (BRNG) 1245 8:30 am to 9:00 am

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Session Chair: Jie Shen

Invited Talk 1 BRNG 1245 9:00 am to 9:45 am

Analysis of Electromagnetic Waves in Random Media

Liliana Borcea

Coffee Break 9:45 am to 10:00 am

Invited Talk 2 BRNG 1245 10:00 am to 10:45 am

Babich's Expansion and the Fast Huygens Sweeping Method for the Helmholtz Wave Equation at High Frequencies

Jianliang Qian

Coffee Break 10:45 am to 11:00 am

Invited Talk 3 BRNG 1245 11:00 am to 11:45 am

Inverse Problems in Quantum Optics

John Schotland

Lunch Break 12:00 pm to 2:00 pm

Session Chair: Plamen Stefanov

Invited Talk 4 BRNG 1245 2:00 pm to 2:45 pm

Inverse Problems for Non-Linear Hyperbolic Equations

Gunther Uhlmann

Coffee Break 2:45 pm to 3:00 pm

Invited Talk 5 BRNG 1245 3:00 pm to 3:45 pm

A Fokker-Planck Equation Inspired Approach for Information Propagation on Networks

Haomin Zhou

Coffee Break 3:45 pm to 4:00 pm

Session Chair: Peijun Li

Invited Talk 6 BRNG 1245 4:00 pm to 4:30 pm

Hyperspectral Synthetic Aperture Radar

Andrew Homan

Invited Talk 7 BRNG 1245 4:30 pm to 5:00 pm

Constructing Travel Times and Redatuming via the Boundary Control Method

Paul Kepley

Invited Talk 8 BRNG 1245 5:00 pm to 5:30 pm

Thermo-Acoustic Tomography in Bounded Domains

Yang Yang

ABSTRACTS

Analysis of Electromagnetic Waves in Random Media

Liliana Borcea
Department of Mathematics, University of Michigan

We study Maxwell's equations in media with small random fluctuations of the electric permittivity, to obtain a detailed mathematical characterization of the statistics of the electric and magnetic fields at long distances of propagation. We introduce a novel wide-angle wave propagation regime, which is mathematically justified by scaling assumptions. In this regime, we obtain a decomposition of the waves in transverse electric and magnetic modes with random amplitudes. These amplitudes account for the cumulative scattering effects in the medium, and satisfy a system of stochastic differential equations which can be analyzed with the Markov limit theorem. The result is an explicit quantification of the randomization of the waves due to scattering, an understanding of polarization effects, and a mathematical justification of the radiative transport equations with polarization.

Babich's Expansion and the Fast Huygens Sweeping Method for the Helmholtz Wave Equation at High Frequencies

Jianliang Qian
Department of Mathematics, Michigan State University

Starting from Babich's expansion, we develop a new high-order asymptotic method, which we dub the fast Huygens sweeping method, for solving point-source Helmholtz equations in inhomogeneous media in the high-frequency regime and in the presence of caustics. The first novelty of this method is that we develop a new Eulerian approach to compute the asymptotics, i.e. the traveltime function and amplitude coefficients that arise in Babich's expansion, yielding a locally valid solution, which is accurate close enough to the source. The second novelty is that we utilize the Huygens–Kirchhoff integral to integrate many locally valid wavefields to construct globally valid wavefields. This automatically treats caustics and yields uniformly accurate solutions both near the source and remote from it. The third novelty is that the butterfly algorithm is adapted to accelerate the Huygens-Kirchhoff summation, achieving nearly optimal complexity $O(N \log N)$, where N is the number of mesh points; the complexity prefactor depends on the desired accuracy and is independent of the frequency.

Inverse Problems in Quantum Optics

John Schotland
Department of Mathematics, University of Michigan

I will review recent work on inverse problems that arise in imaging with entangled states of light. Experiments involving entanglement via post-detection selection or due to illumination with down converted photons will be analyzed.

Inverse Problems for Non-Linear Hyperbolic Equations

Gunther Uhlmann
Department of Mathematics, University of Washington

We consider inverse problems for the Einstein equation with a time-depending metric on a 4-dimensional globally hyperbolic Lorentzian manifold. We formulate the concept of active measurements for relativistic models. We do this by coupling Einstein equations with equations for scalar fields.

The inverse problem we study is the question of whether the observations of the solutions of the coupled system in an open subset of the space-time with the sources supported in this set determines the properties of the metric in a larger domain. To study this problem we define the concept of light observation sets and show that knowledge of these sets determine the conformal class of the metric. This corresponds to passive observations from a distant area of space which is filled by light sources.

We will start by considering inverse problems for scalar non-linear hyperbolic equations to explain our method. No previous knowledge of Lorentzian geometry or general relativity will be assumed. This is joint work with Y. Kurylev and M. Lassas.

A Fokker-Planck Equation Inspired Approach for Information Propagation on Networks

Haomin Zhou
School of Mathematical Sciences, Georgia Insitute of Technology

As information, such as viral signals or and infectious diseases, spreads on various kinds of real-world networks, we are interested in estimating and predicting their influences which, for example, are usually defined as the expected number of individuals involved in the spread, up to a target time. Methods characterizing continuous-time propagation adaptively model the dynamics of such spreads. However, they tend to encounter significant computational challenges in influence prediction which is a fundamental problem in many applications. In this presentation, we introduce a new analysis framework, based on the discrete Fokker-Planck equations, and associated computational methods to tackle this problem. It gives rise to a class of novel algorithms that work effectively for networks with unknown propagation models or parameters, or even unobserved network structure. Numerical tests are carried out on several synthetic and real-world networks to show the promising performance of the proposed method. I will also briefly address the connection between our new model to the optimal transport theory. This is based on a recent work with Shui-Nee Chow (Math, GT), Xiaojing Ye (Math, Georgia State), and Hongyuan Zha (CSE, GT).

Hyperspectral Synthetic Aperture Radar

Andrew Homan
Matrix Research

Typical synthetic aperture radar (SAR) imaging techniques neglect frequency-dependent dispersive effects. We propose an imaging algorithm, hyperspectral synthetic aperture radar (HSAR) which forms an image of the complex-valued scene reflectivity function as it depends on (x, y, frequency), or equivalently, (x, y, time delay). The algorithm permits arbitrary flight paths and arbitrary waveforms. We provide some numerical examples illustrating the approach. This work was done jointly with Margaret Cheney and Matthew Ferrara.

Constructing Travel Times and Redatuming via the Boundary Control Method

Paul Kepley
Department of Mathematics, Purdue University

We consider two problems which have applications to the inverse boundary value problem for a Riemannian wave equation with a smooth unknown metric. For data we use the Neumann-to-Dirichlet (N-to-D) map, and our main theoretical tool is a geometric variant of the boundary control (BC) method. First, we present a technique to use the N-to-D map to construct travel times between interior points (belonging to a semi-geodesic neighborhood of the boundary) and boundary points. Next we consider a redatuming problem. In this problem, the metric is known in a near boundary region, and the objective is to use the N-to-D map and metric near the boundary to synthesize receiver measurements for waves generated by internal sources located there. We provide computational experiments demonstrating both procedures.

Thermo-Acoustic Tomography in Bounded Domains

Yang Yang
Department of Mathematics, Purdue University

Thermo-Acoustic tomography (TAT) is a recently developed coupled physics imaging modality. In this talk we discuss the mathematical model of TAT with the presence of sound-hard reflectors. We propose an averaged sharp time reversal algorithm which leads to an exponentially convergent Neumann series reconstruction. Numerical implementation of the algorithm is presented and compared with the result by the Landweber iterative algorithm. This is joint work with Plamen Stefanov.
