# MA265 Linear Algebra — Practice Exam 1

*Date:* Spring 2021 *Duration:* 60 min

**Name:** ______________________________

**PUID:** ______________________________

- All answers must be justified and you must show all your work in order to get credit.
- The exam is open book. Each students should work independently, Academic integrity is strictly observed.

| **Problem** | **Points** | **Score** |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| **Total:** | 100 | |

**1.** Consider the matrices $A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Let $S$ be the subspace of $\mathbb{R}^3$ consisting of those vectors $\mathbf{x}$ such that $A\mathbf{x} = B\mathbf{x}$. Find a basis of $S$. [10pt]

**2.** Consider the vectors [10pt]

$$\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $A$ be a $3 \times 3$ matrix such that $A\mathbf{u} = \mathbf{0}$, $A\mathbf{v} = \mathbf{0}$ and $A\mathbf{w} = \mathbf{w}$. What is the rank of $A$?

**3.** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear map such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and [10pt]

$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. Compute $T\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right)$.

**4.** Let $A$ be a $3 \times 5$ matrix. Which of the following statements are true? Indicate clearly **all** correct answers. [10pt]

A. The rank of $A$ is 3.

B. The null space of $A$ has dimension at least 2.

C. $A\mathbf{x} = \mathbf{0}$ has only one solution, the trivial solution.

D. There exists two linearly independent vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^5$ such that $A\mathbf{u} = A\mathbf{v} = \mathbf{0}$.

E. The columns of $A$ are linearly dependent.

**5.** Suppose that $A$ and $B$ are $2 \times 2$ matrices satisfying $\det(B) = 8$ and $A^3 = B^2$. Determine the value of $\det(3\, A^T\, BA^{-1}B^{-1}A)$. [10pt]

**6.** Consider the matrix $A = \begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & 2 & 10 & 0 \\ 0 & 3 & 1 & 0 \\ 1 & 0 & 0 & 5 \end{pmatrix}$. Compute the $(3, 2)$ entry of the adjugate matrix $\mathrm{adj}(A)$. [10pt]

**7.** Consider the matrices [10pt]

$$A = \begin{bmatrix} a & b & c & d \\ x & y & z & 0 \\ -3 & 7 & 2 & 11 \\ -1 & 1 & 2 & 10 \end{bmatrix}, \quad B = \begin{bmatrix} x & y & z & 0 \\ -3+bx & 7+by & 2+bz & 11 \\ a & b & c & d \\ -1 & 1 & 2 & 10 \end{bmatrix}.$$

Suppose that $\det(A) = 3$. Find $\det(2B)$.

**8.** Consider a linear system whose augmented matrix is of the form [10pt]

$$[A|\vec{b}] = \left[\begin{array}{ccc|c} 1 & 0 & -2 & a \\ 0 & 1 & a & a-3 \\ 0 & 0 & a-4 & a-3 \end{array}\right]$$

(i) For what values of $a$ will the system have no solution?

(ii) For what values of $a$ will the system have a unique solution?

(iii) For what values of $a$ will the system have infinitely many solutions?

**9.** Consider the system: [10pt]

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + z &= 9 \\ x + y + (a^2 - 5)z &= a \end{aligned}$$

For which value of $a$ does the system have infinitely many solutions?

**10.** Find a subset $T$ of the set $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right\}$ such that $T$ is a basis for the subspace of $\mathbf{R}^3$ spanned by $S$. [10pt]