Functional Analysis, Spring 2020
Homework # 3
due on Wednesday February 5 in class

1. Let X be a normed linear space whose dual X^* is finite dimensional. Show that X is finite dimensional.

2. Let X be an infinite dimensional normed linear space. Show that there is a linear map $f : X \to \mathbb{R}$ which is not continuous.
(*Hint:* Use an algebraic basis consisting of norm-one vectors of the linear space X to construct an unbounded linear map $f : X \to \mathbb{R}$.)

3. Let X be a normed space and let Y be a linear subspace of X. Let $\pi : X \to X/Y$, $\pi(x) = \bar{x} = x + Y$ be the quotient map. On the quotient linear space X/Y of cosets define

$$\|\bar{x}\|_{X/Y} = \inf\{\|x - y\|_X : y \in Y\}.$$

(a) Show that $\bar{x} \mapsto \|\bar{x}\|_{X/Y}$ is a seminorm and that it is a norm if and only if Y is closed in X.
(b) Show that if X is a Banach space and if Y is a closed linear subspace of X then X/Y is a Banach space with respect to the norm defined above.
(*Hint*: One may use the result from question #2 from HW 1.)