Functional Analysis, Spring 2020
Homework # 3
due on Wednesday February 12 in class

1. Show that any finite dimensional linear subspace of a normed linear space is closed.

2. Let X be an infinite dimensional Banach space. Show that any algebraic basis of the vector space X is uncountable.

(Hint: Seeking a contradiction assume X has a countable basis. Then one may use #1 and Baire's Category Theorem.)

3. We have seen in HW2 that the linear map $J : \ell^1(\mathbb{N}) \to \ell^2(\mathbb{N})$, defined by $J(\xi) = \xi$ is continuous. Show that the range of J is not closed. In other words show that the subspace $J(\ell^1(\mathbb{N}))$ of $\ell^2(\mathbb{N})$ is not a closed subset of of $\ell^2(\mathbb{N})$.

4. Let X be the normed space of all analytical functions from $\mathbb{C}$ to $\mathbb{C}$, with norm

$$\|f\| = \sup_{|z|=1} |f(z)|.$$

Let $T : X \to X$ be defined by $T(f)(z) = f(z/2)$. Show that T is a linear continuous bijection but T^{-1} is not bounded.