Homework 1 Solutions

You should use the following solutions to grade your work. Give 10 points per problem. Deduct 1 point for each omitted or incorrect axiom or property. Deduct 1 point if the Assume statement is missing. Deduct 2 points for not checking the answer in Problem 1. (See the discussion following Example 1.)

Solution to 1:

Assume that x satisfies the given equality. Then

$$\begin{aligned}
7x + (-5) &= 19 \quad \text{Def. of Subtraction}\\
(7x + (-5)) + 5 &= 19 + 5 \quad \text{(A0)}\\
7x + ((-5) + 5) &= 24 \quad \text{(A1)}\\
7x + (5 + (-5)) &= 24 \quad \text{(A4)}\\
7x + 0 &= 24 \quad \text{(A3)}\\
7x &= 24 \quad \text{(A2)}\\
7^{-1}(7x) &= 7^{-1}24 \quad \text{(M0),(M3)}\\
(7^{-1}7)x &= 24 \cdot 7^{-1} \quad \text{(M1)}\\
(7 \cdot 7^{-1})x &= 24 \cdot 7^{-1} \quad \text{(M4)}\\
1x &= \frac{24}{7} \quad \text{(M3), Def. of Division}\\
x &= \frac{24}{7} \quad \text{(M4),(M2)}
\end{aligned}$$

Conversely, if $x = \frac{24}{7}$,

$$7x - 5 = 7\frac{24}{7} - 5 = 24 - 7 = 19$$

showing that $x = 24/7$ does indeed solve the equality. □

Solution to 3:

Let x,y,z,and w be real numbers. Then

$$\begin{aligned}
(x+y)(z+w) &= x(z+w)+y(z+w) \quad \text{(C1)}\\
&= (xz+xw)+(yz+yw) \quad \text{(D)}\\
&= xz+(xw+(yz+yw)) \quad \text{(A1)} \quad a=xz; b=xw; c=yz+yw\\
&= xz+((xw+yz)+yw) \quad \text{(A1)} \quad a=xw; b=yz; c=yw\\
&= xz+((yz+xw)+yw) \quad \text{(A4)}\\
&= xz+(yz+(xw+yw)) \quad \text{(A1)} \quad a=yz; b=xw; c=yw\\
&= (xz+yz)+(xw+yw) \quad \text{(A1)} \quad a=xz; b=yz; c=xw+yw
\end{aligned}$$

Solution to 5:

Let x,y,z,and w be numbers. Then

$$\begin{aligned}
x+(y+z+w) &= x+((y+z)+w) \quad \text{(Definition 2)}\\
&= x+(y+(z+w)) \quad \text{(A1)} \quad , a=y, b=z, c=w\\
&= (x+y)+(z+w) \quad \text{(A1)} \quad , a=x, b=y, c=z+w
\end{aligned}$$

which proves the first equality asked for.

To prove the second, we continue this chain of equalities as

$$\begin{aligned}
(x+y)+(z+w) &= ((x+y)+z)+w \quad \text{(A1)} \quad , a=x+y, b=z, c=w\\
&= (x+y+z)+w \quad \text{(Definition 2)}
\end{aligned}$$

which is the second equality asked for.

Solution to 11:

$$\begin{aligned}
2\cdot 3 &= 2\cdot(2+1) \quad \text{Def. of 3}\\
&= 2\cdot 2+2\cdot 1 \quad \text{D}\\
&= 4+2 \quad \text{Example 6, M2}\\
&= 4+(1+1) \quad \text{Def. of 2}\\
&= (4+1)+1 \quad \text{A1}\\
&= 5+1 \quad \text{Def. of 5}\\
&= 6 \quad \text{Def. of 6}
\end{aligned}$$

Solution to 12:

Suppose that

$$c+d=0.$$

Then

$$\begin{aligned}(c+d)+(-d) &= 0+(-d) \quad \text{A0,A3}\\ c+(d+(-d)) &= (-d)+0 \quad \text{A1,A4}\\ c+0 &= -d \quad \text{A3,A2}\\ c &= -d \quad \text{A2}\end{aligned}$$

as desired.