MA 301 Test 4, Spring 2006

*2 hours, calculator allowed, no notes. Provide paper for the students to do work on. Students should not write answers on test sheet.*

**All answers must be justified.** Simply stating an answer is not worth any credit.

(1) State the "official" definition of "$\lim_{x\to a} f(x) = L$." 8 pts

(2) Show that it is impossible to list all numbers in the interval $(0, 1)$. How does this prove that the set in question is uncountable. 12 pts

(3) Let $A$ be the set of all rational numbers in the open interval $(0, 1)$ and let $B$ be the set of all rational numbers in the closed interval $[0, 1]$. Describe an explicit one-to-one correspondence between $A$ and $B$. 10 pts

(4) For the following function, find a value of $a$ for which $f(x)$ is continuous at $x = 1$ 10 pts

$$f(x) = \begin{cases} (x+a)^2 & x < 1 \\ 3 + x & x \geq 1 \end{cases}$$

(5) Use a $\delta$-$\epsilon$ argument to prove that 12 pts

$$\lim_{x\to 2} \frac{2x+1}{x+1} = \frac{5}{2}.$$

(6) Use a $\delta$-$\epsilon$ argument to prove that 12 pts

$$\lim_{x\to 1} \sqrt{3x+1} = 2.$$

(7) Use a $\delta$-$\epsilon$ argument to prove that 12 pts

$$\lim_{x\to 2} \frac{1}{3-x} = 1.$$

(8) Assume that $\lim_{x\to a} f(x) = 1$. Use a $\delta$-$\epsilon$ argument to prove that 12 pts

$$\lim_{x\to a} \frac{f(x)}{f(x)+2} = \frac{1}{3}.$$

(9) Use a $\delta$-$\epsilon$ argument to prove Theorem 3 on p. 164 of the notes: 12 pts