Math 462 Fall

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Sample Final Questions

(1) Give the definition of the tangent plane T_pS to a point p of a regular surface S.

(2) Let $\phi : S_1 \to S_2$ be a differentiable map between two regular surfaces. How is the differential $d\phi(p) : T_pS_2 \to T_{\phi(p)}S_2$ defined.

(3) When is a regular surface orientable? Give examples of orientable and non-orientable surfaces. Explain your examples.

(4) Give the definition of the first fundamental form.

(5) Calculate the first fundamental form of the upper sheet of the hyperboloid $x^2 + y^2 - z^2 = 1$.

(6) What is the Gauss map? Give the Gauss map for the cylinder, a graph and the sphere.

(7) Give the definition of the second fundamental form.

(8) Caclulate the 2nd fundamental form for a graph.

(9) Give a geometric interpretation of $II_p(v)$.

(10) Derive the Euler formula. Let e_1, e_2 be a basis of $T_p(S)$ such that e_1 and e_2 are principal curvature directions of curvature k_1, k_2. Calculate the normal curvature of the curve α through $p = \alpha(0)$ if $\alpha'(0) = cos(\theta)e_1 + \sin(\theta)e_2$.

(11) Give the definition of Gauss curvature and mean curvature.

(12) Calculate the Gauss and mean curvature of the sphere and the cylinder of radius one.

(13) State Gauss's Theorema Egregium.

(14) Give an expression for K in terms of E, F, G, e, f, g (not using their derivatives).

(15) The cylinder is isometric to the plane.

(16) Give the definition of

- ▷ the Chistoffel symbols Γ_{ij}^{k}.
- ▷ The covariant derivative of a vector field along a curve
- ▷ A parameterized geodesic.
- ▷ A geodesic.
- ▷ Geodesic curvature.

(17) Show that great circles are geodesics and other latitudes are not.

(18) State Bonnet's Theorem about the existence of surfaces.

(19) State the Theorem/relation of Clairault.

(20) State the local and global version of the Gauss–Bonnet theorem. Be sure to say what each symbol stands for.

(21) Give the definition of an abstract surface. Give examples of surfaces which are abstract, but not regular surfaces.

(22) Give a version of the hyperbolic plane as an abstract surface.

(23) True (T) or false (F).

- ▷ A surface always has a unique orientation.
- ▷ If a surface has an orientation it is unique. The tangent plane at a point of a regular surface is a vector space
- ▷ of dim 2 (why?).
- ▷ The Gauss curvature is always positive.
- ▷ The mean curvature and the Gauss curvature are independent.
- ▷ The coefficients of the first and second fundamental form are independent.
- ▷ The Gauss curvature depends on the embedding.
- ▷ Principal curvature directions are perpendicular.

- ▷ The maximum and minimum normal curvature are given by the principal curvatures.
- ▷ The normal curvature of a curve of a regular surfaces at a point p only depends on its tangent at p.
- ▷ The curvature of a curve of a regular surfaces at a point p only depends on its tangent at p.
- ▷ There is a regular surface of constant Gauss curvature -1.
- ▷ If a surface has constant Gauss curvature 0 it is a part of a plane and if it has constant Gauss curvature 1 it is a part of a sphere of radius 1.
- ▷ There is a map of the $S^2 - \{S, N\}$ which renders correct distances and angles (why?).
- ▷ Conformal maps never preserve areas.
- ▷ There is a surface with E=F=0, G=1, e=g=1, f=0.
- ▷ The Gauss equation depends on the coefficients of the 2nd fundamental form.
- ▷ Local conformal maps that preserve area are local isometries.
- ▷ Loxodromes are geodesics.