

Math 572
Spring 19
Take-home final
Faculty: R. Kaufmann

Name:

Signature:

Student ID Number:

Directions: Please send a pdf (scan) file to rkaufman@math.purdue.edu with a cc to kaufmann.ralph@gmail.com by Wed 05/01 9 pm.

Use this page as a cover page.

Problem 1: a) Give the axioms of a homology theory. b) How are they related to the axioms of a cohomology theory.

Problem 2: Are the chain groups of singular and simplicial chains free? If so give a basis.

Problem 3: Prove that the singular homology groups are functorial.

Problem 4: Give a sketch of the proof that for a triangulable space the singular and simplicial homology are isomorphic.

Problem 5: What is the technical role of \mathcal{A} -small simplices in singular homology. What property of simplices is being used.

Problem 6: How is the chain complex of a CW complex defined. Give the chain groups and a definition of the differential. Also, are the chain groups free? If so give a basis.

Problem 7: Give an overview on how to obtain the Mayer-Vietoris sequence.

Problem 8: Describe the steps in the Künneth formulas for (co)-homology topological spaces. What are the roles of the Alexander-Whitney maps and the Eilenber-Zilber maps.

Problem 9: Calculate the homology and cohomology of the torus, the Klein bottle and $\mathbb{R}P^2$ with $\mathbb{Z}/2\mathbb{Z}$ coefficients using the univesal coefficient theorems.

Problem 10: Compute the homology and the cohomology ring of $T^n = (S^1)^{\times n}$.