

Math 225 Fall 05

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Sample Final Questions

- (1) Give the definition of the tangent plane $T_p S$ to a point p of a regular surface S .
- (2) Let $\phi : S_1 \rightarrow S_2$ be a differentiable map between two regular surfaces. How is the differential $d\phi(p) : T_p S_1 \rightarrow T_{\phi(p)} S_2$ defined.
- (3) When is a regular surface orientable? Give examples of orientable and non-orientable surfaces. Explain your examples.
- (4) Give the definition of the first fundamental form.
- (5) Calculate the first fundamental form of the upper sheet of the hyperboloid $x^2 + y^2 - z^2 = -1$.
- (6) What is the Gauss map? Give the Gauss map for the cylinder, a graph and the sphere.
- (7) Give the definition of the second fundamental form.
- (8) Give a geometric interpretation of $II_p(v)$.
- (9) Derive the Euler formula. Let e_1, e_2 be a basis of $T_p(S)$ such that e_1 and e_2 are principal curvature directions of curvature k_1, k_2 . Calculate the normal curvature of the curve α through $p = \alpha(0)$ if $\alpha'(0) = \cos(\theta)e_1 + \sin(\theta)e_2$.
- (10) Give the definition of Gauss curvature and mean curvature.
- (11) Calculate the Gauss and mean curvature of the sphere and the cylinder of radius one.
- (12) State Gauss's Theorema Egregium.
- (13) State Bonnet's Theorem about the existence of surfaces.
- (14) Give the definition of an abstract surface. Give examples of surfaces which are abstract, but not regular surfaces.

(15) Give a version of the hyperbolic plane as an abstract surface.

(16) True (T) or false (F).

- ▷ A surface always has a unique orientation.
- ▷ If a surface has an orientation it is unique.
- ▷ The tangent plane at a point of a regular surface is a vector space of dim 2 (why?).
- ▷ The Gauss curvature is always positive.
- ▷ The mean curvature and the Gauss curvature are independent.
- ▷ The coefficients of the first and second fundamental form are independent.
- ▷ The Gauss curvature depends on the embedding.
- ▷ Principal curvature directions are perpendicular.
- ▷ The maximum and minimum normal curvature are given by the principal curvatures.
- ▷ The normal curvature of a curve of a regular surfaces at a point p only depends on its tangent at p .
- ▷ The curvature of a curve of a regular surfaces at a point p only depends on its tangent at p .
- ▷ There is a regular surface of constant Gauss curvature -1 .
- ▷ If a surface has constant Gauss curvature 0 it is a part of a plane and if it has constant Gauss curvature 1 it is a part of a sphere of radius 1.
- ▷ There is a map of the $S^1 - \{S, N\}$ which renders correct distances and angles (why?).