

Math 225 Fall 05

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Sample Midterm Questions

- (1) Give the definition of a regular parameterized differentiable curve.
- (2) Give the definition of the arc length parameter.
- (3) Can every curve be reparameterized by arc length?
- (4) Give an equation for the tangent line of a regular curve $\alpha : I \rightarrow \mathbb{R}^3$ at the point $t \in I$.
- (5) What can you say about a curve α whose second derivative $\alpha'' = 0$.
- (6) Give a parameterization of a circle of radius a clockwise/counterclockwise
- (7) Give a parameterization of a helix of radius a with height c .
- (8) Let α be a curve parameterized by arc length. Define the tangent, normal and bi-normal vectors \mathbf{t} , \mathbf{n} and \mathbf{b} of α at $\alpha(t)$. Do you need any additional assumptions for this? What would happen if the curve were not parametrized by arc length?
- (9) Define curvature and torsion.
- (10) What is the torsion of a planar curve?
- (11) Write down the Frenet formulas.
- (12) State the fundamental theorem of the local theory of curves.
- (13) Give a curve which has constant curvature and torsion.
- (14) Is there a curve $\alpha : [-1, 1] \rightarrow \mathbb{R}^3$ whose curvature is $\sin(t)$ and torsion $\cos(t)$, how about curvature t^2 and torsion t^3 .
- (15) True or false. To first order a curve lies in a plane. To third order a curve lies in a half space. To second order the curve lies in the normal plane. The osculating plane is defined to be the plane containing the tangent and the normal vector.
- (16) Define a regular surface
- (17) Give examples of regular surfaces.
- (18) Give coordinate charts which cover S^2 .
- (19) Give the definition of a rotational surface. Is a rotational surface regular? Give coordinate patches which cover a given a rotational surface.
- (20) True or false. Every regular surface has a global parameterization. One of the projections of a regular surface to the three coordinate planes is a graph. Locally a surface is a graph. The inverse image of a regular value is a regular surface. The inverse image of a critical value is necessarily not regular.