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Math 337, Fall 2006

Homework 1

Directions This is part homework and part review, there is also a section of extra problems, where you have to read on your own. Good references (beside your notes) are

(1) Hirzebruch, Friedrich *Topological methods in algebraic geometry.* Classics in Mathematics. Springer-Verlag, Berlin, 1995. xii+234 pp
(2) Hirzebruch, Friedrich; Berger, Thomas; Jung, Rainer *Manifolds and modular forms.* Aspects of Mathematics, E20. Friedr. Vieweg & Sohn, Braunschweig, 1992. xii+211 pp.

Review

Problem 1: Define the cobordism ring. Give a ring that is isomorphic to the cobordism ring over $\mathbb{Q}$.

Problem 2: Give the relations between a genus ϕ, its power series Q and the logarithm of the power series g. What is the significance of g?

Problem 3: Give the relation between the power series Q and the set of multiplicative polynomials K.

Problem 4: Define the $\hat{A}$ genus, the L-genus and the elliptic genera.

Problem 5: How does one obtain elliptic genera from lattices in $\mathbb{C}$?

Problem 6: Define the χ_y genus and discuss its specializations to $y = -1, 0, 1$.

Problem 7: Define the equivariant signature of the loop space and the Witten genus.

Problem 8: State the Atiyah-Singer Index theorem.

Problems

Problem 9: Calculate the total Chern class of $\mathbb{CP}^n$

Problem 10: Calculate the total Pontrjagin class of $\mathbb{HP}^n$

Problem 11: Prove that the multiplicative polynomials from a given power series are multiplicative. I.e. if $Q(x)$ gives the $K_n(p_1, \dots p_n)$ then show that if

$$1 + p_1 + p_2 + \dots = (1 + p_1' + p_2' + \dots)(1 + p_1'' + p_2'' + ...)$$
$$\sum_{n\geq 0} K_n(p_1, \dots, p_n) = \sum_{n\geq 0} K_n(p_1', \dots, p_n') \cdot \sum_{n\geq 0} K_n(p_1'', \dots, p_n'')$$

PROBLEM 12: Prove the equivalence of the tree conditions for an elliptic genus (1) differential equation (2) addition theorem and (3) duplication formula.

EXTRA PROBLEMS

EXTRA PROBLEM 1: State the equivariant Atiyah-Singer index theorem.

EXTRA PROBLEM 2: Define the Dirac operator and give its relation to the $\hat{A}$-genus.