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**Math 337, Fall 2006**

HOMEWORK 2

**Directions:** This is part homework and part review. Good references (beside your notes) are

1. Gelfand, Sergei I.; Manin, Yuri I. Methods of homological algebra. Second edition. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xx+372 pp.
2. Kassel, Christian Quantum groups. Graduate Texts in Mathematics, 155. Springer-Verlag, New York, 1995. xii+531 pp
3. Atiyah, Michael Topological quantum field theories. Inst. Hautes Études Sci. Publ. Math. No. 68 (1988), 175–186 (1989).
4. Markl, Martin; Shnider, Steve; Stasheff, Jim Operads in algebra, topology and physics. Mathematical Surveys and Monographs, 96. American Mathematical Society, Providence, RI, 2002. x+349 pp.

**Review:**

PROBLEM 1: Define the notions of tensor category and monoidal functor. What does it mean that these structures are strict.

PROBLEM 2: Define a Topological field theory in terms of categories and functors.

PROBLEM 3: Define a Hopf algebra. Give all the structures for  $k[G]$ .

PROBLEM 4: What are quasi-Bialgebras and braided quasi-Bialgebras? What is another name for the latter?

PROBLEM 5: Define an operad. Give two examples.

PROBLEM 6: Define the notions of a cyclic and a modular operad.

PROBLEM 7: Define the Hochschild complex for an associative algebra.

**Problems:**

PROBLEM 8: Show that a TFT yields a Frobenius algebra. What are the basic operations and the appropriate surfaces. What are the relations and the relevant surfaces/decompositions.

PROBLEM 9: Prove that for a bi-algebra the following two statements are equivalent: (i)  $\mu, \eta$  are morphisms of co-algebras, (ii)  $\Delta, \epsilon$  are morphism of algebras.

PROBLEM 10: Show that if  $H$  is a commutative, co-commutative Hopf algebra  $S^2 = id_H$

PROBLEM 11: Show that a cyclic operad is equivalent to a set of correlations functions on an algebra with a symmetric pariring.

PROBLEM 12: What are the operads for commutative; associative; commutative associative,  $A_\infty$  and Frobenius algebras?