Analysis and Control of the Korteweg-de Vries Equation

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ABSTRACT

In this talk we will discuss the KdV equation posed on a bounded domain with either a periodic boundary condition or Dirichlet boundary conditions:

$$\begin{cases} u_t + u_x + uu_x + u_{xxx} = f, \qquad x \in (0,L),\ t \in (0,\infty), \\ u(x,0) = \phi(x), \\ u(0,t) = u(L,t), \qquad u_x(0,t) = u_x(L,t), \qquad u_{xx}(0,t) = u_{xx}(L,t). \end{cases} \tag{1}$$

and

$$\begin{cases} u_t + u_x + uu_x + u_{xxx} = f, \qquad x \in (0,1),\ t \in (0,\infty), \\ u(x,0) = \phi(x), \\ u(0,t) = h_1(t), \qquad u(L,t) = h_2(t), \qquad u_x(L,t) = h_3(t). \end{cases} \tag{2}$$

Considering $f = f(x,t)$, $h_j(t),\ j = 1,2,3$ as control inputs, we will demonstrate both control systems (1) and (2) are exactly controllable. The various smoothing properties of the KdV equation will be discussed and be demonstrated that how those smoothing properties will play an indispensable role in establishing these exact controllability.