# Water-waves as a spatial reversible dynamical system, influence of an essential spectrum

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The mathematical study of travelling waves, in the context of two dimensional potential flows in one or several layers of perfect fluid(s), in the presence of free surface and interfaces can be set as an ill-posed evolution problem, where the *horizontal space variable plays the role of a "time"*.

In the finite depth case, the spectrum of the linearized operator is only composed with isolated eigenvalues of finite multiplicities, with a finite number near the imaginary axis. The study of near equilibria waves reduces to a low dimensional center manifold, leading to a *reversible ordinary differential equation*. A normal form analysis shows that in most cases, the dynamics on the center manifold is the one of a *perturbed integrable system*, where all types of solutions are known. We shall give an idea of the method and review typical results.

A case of greater physical interest is the infinite depth limit. In such a case, the above reduction technique fails because the linearized operator possesses an *essential spectrum filling the whole real axis*, and new adapted tools are necessary. We give the method and results for two types of systems. The first type is when the dominant part of the bifurcating solutions is provided by the critical eigenvalues near the imaginary axis - an example is with a single infinitely deep layer, with surface tension at the free surface, which leads to solitary waves with polynomially decaying oscillations at infinity. The second type is when the dominant part of the bifurcating solutions is provided when a pair of imaginary eigenvalues merge at 0, which is part of the essential spectrum, and disappear when a papameter is varying. An example is with two superposed layers, the bottom one being infinitely deep, with no surface tension at the interface and strong enough surface tension

at the free surface. In this later case we obtain a family of solitary waves of Benjamin-Ono type (hence decaying polynomially at infinity).

In our lecture we give quite general assumptions on infinite-dimensional reversible systems for this types of bifurcations in presence of essential spectrum.

**Bibliography**

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