In order to prepare for midterm 1:

- Go over all HW problems that you did not get full marks on. Redo them so that you would get full marks on them. For each problem you did not get full marks, do one more example of that sort.
- Do the same with all quiz questions.
- Do the sample problems below. Warning: These are somewhat more involved than the problems on the midterm will be. You will be learning things while you are doing these problems. The midterm questions should seem easier.
- If you are worried, ask a friend to pick 12 HW type questions from the chapters 12 and 13, in a representative way. Sit down for an hour and do them under test conditions. (No phone, no help, no notes, no book). Thencheck what you did not know. Do 3 problems for each question your friend picked that you could not answer and do them. Repeat the procedure.

1. In the equation $x^2 \pm 5y^a \pm 21z^b = c$, we allow all possible choices of plus and minus, and for a, b being either 1 or 2, and for c being either 1 or 0. Making random choices (flip a coin) for all 5 parameters, sketch the surfaces to the equation in at least 4 cases.
2. Discuss the geometric/physical meaning of tangent, normal, binormal, curvature, torsion, kissing plane, kissing circle.
3. Determine whether the line of intersection of $3x - y + 2z = 4$ and $2x + y - z = 1$ meets the plane $x - y - z = 3$. In general, given 3 planes, devise a quick test that checks whether the intersection of all three planes is empty or not. Be careful not to forget special cases.
4. Consider the curve $(x, y, z) = (t, t^2, t)$. Compute T, N, B. Where does this curve have maximal curvature, and where maximal torsion?
5. Find the point P on the cone $4x^2 = y^2 + z^2$ that is closest to $Q = (1, 2, 3)$. (Hint: first, rotate so that Q lies in the xz-plane. Let Q' be the rotated Q, P' the rortated P. Then use that the line from P to Q is perpendicular to the surface to find a linear equation that P' must satisfy. Use it and the cone equation to find a simple quadratic relation that the x and z coordinate of P' must satisfy. Factor it. You get 2 possible linear relations between x and z for the optimal P'. In each case we now have 2 linear equations for P'. That is, you found two lines, on one of which P must sit. Now find the best P' for each line with known methods. Finally, rotate back.)
6. Find the angle between the sides of a regular tetrahedron. (The angle between the *edges* is obviously 60 degrees, so that is not what I am asking).
7. Find the surface area of the tetrahedron with vertices $(1, t, t^2)$ for $t = 0, 1, 2, 3$.

8. A boat with broken rudder is drifting down the Mississippi, hoping to get to the port in Natchez. At noon, the boat is 2 km North of the port, and 0.3 km West. The current leads it straight south at 13 m per second. The boat also has a sail up. There is a gentle wind blowing from the Northwest, at 5 m per second. The sail catches 60 percent of the wind speed. In order to make it to the port, should the people on the boat row forward or backward? And at what speed relative to the water?

9. Let $f(x, y, z)$ describe a surface in 3-space that contains the point P. Explain why one should not always hope for a "kissing sphere" to S at P, although curves always have a kissing circle. (The best way is to check through some basic examples of quadric surfaces and see whether such kissing sphere would make sense). If you can find such example, devise a test which determines whether such kissing sphere exists.

10. Suppose S is a quadric surface in 3-space. Suppose also that L is any line that is not completely inside S. How many points can the intersection $S \cap L$ have? For each possible answer, describe one example.

11. A ball is thrown (at time t=0) at an angle of 30 degrees upwards against the horizontal, with speed 32 m/s. Assuming a level playing field, describe location and speed in terms of t. Compute total travel distance. Find another angle that makes the ball land in the same spot, assuming the same initial speed.