Sample problems for midterm 2.

1. Determine for $\sum_{n=2}^{\infty} \frac{4 \cdot 6 \cdot 8 \cdots (2n)}{5^{n+1}(n+2)!} x^n$ the ROC, and test whether the series converges at the two endpoints.

2. Approximate $\int_0^x \sin(t^2) dt$ by a polynomial on the interval $[0, 1]$ to within $1/1000$.

3. Determine whether $\frac{x^4}{x^4+y^2}$ has a limit at $x = y = 0$. If so, find the linit.

4. Find the tangent plane at $P = (1, 1, 2)$ for the surface $x^2 + y^2 = z$.

5. Two resistors $R_1 = 5000\Omega$ and $R_2 = 1000\Omega$ are in a parallel connection. Using differentials, which change in the resistors will produce a greater change in the overall resistance: increasing R_1 by 20Ω or R_2 by 1Ω?

   (Recall: parallel resistance satisfies $1/R = 1/R_1 + 1/R_1$.)

6. What kind of critical point does $f = x^2 + kxy + y^2$ have at $x = y = 0$, if k is some constant?

7. Find the maximum of $f = ab^2c^3$ if $a, b, c \geq 0$ are constrained by $a + b + c - 3 = 0$.

8. Let $w(x, y, z) = \cos(xy) \cdot \sin(yz) + xyz$. Suppose x, y, z are constrained to the surface $x^2 + 4y^2 + 9z^2 = 1$. Compute $(\partial z/\partial y)_x$ and $(\partial w/\partial y)_x$ in the point $(x, y, z) = (1/3, 1/3, 2/9)$.