

Homework 5

Due on Oct 6th before 10am on gradescope.

1. (20 pts) Let $a_n \geq 0$ be decreasing and assume $\sum_{n=1}^{\infty} a_n$ converges. Prove $na_n \rightarrow 0$. (See Problem 7-1 on Page 111).

2. (20 pts) Page 111, Problem 7-2. Prove that if $|a_{n+1}/a_n| \leq |b_{n+1}/b_n|$ for $n \gg 1$, and $\sum b_n$ is absolutely convergent, then $\sum a_n$ is absolutely convergent.

3. (20 pts) Page 112, Problem 7-4. Prove that

$$\lim |a_{n+1}/a_n| = L \Rightarrow \lim |a_n|^{1/n} = L.$$

4. (40 pts) Page 110, Exercise 7.4-7.5: 1 (a)(c)(e)(g)

Hint: you can use the inequality $\sin x \geq x/2$ if $x > 0$ is small enough.