Homework 6

Due on Oct 13 before 10 am on gradescope.

1. (30 pts) Determine the convergence of the series and explain why:

   (a) $\sum\limits_{n=2}^{\infty} \frac{1}{n \ln n}$

   (b) $\sum\limits_{n=1}^{\infty} \frac{2^n n!}{n^n}$

   (c) $\sum\limits_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^2}$

   Hint:

   (a) you can find the antiderative of $\frac{1}{x \ln x}$ by change of variables;

   (b) In section 1.5, we have learned $(1+1/n)^n \to e = 2.7182...$, you can use this fact (can you prove this fact using only theorems in first six chapters?):

   $$(1+1/n)^n \to e \Rightarrow (1-1/n)^n \to 1/e.$$

   (c) You can use this fact:

   $$(1+r/n)^n \to e^r,$$

   where r is a real number.

2. (20 pts) Prove that a conditionally convergent series has infinitely many positive terms and infinitely many negative terms.

3. (20 pts) Page 111, Exercise 7.7/1. Given a series $\sum a_n$, define a new series $\sum b_k$ where

   $$b_0 = a_0 + \cdots + a_{n_0}, b_1 = a_{n_0+1} + \cdots + a_{n_1}, b_2 = a_{n_1+1} + \cdots + a_{n_2}, \cdots$$

   Prove that if $\sum a_n$ converges, so does $\sum b_k$, and to the same sum.

4. (30 pts) Page 123, Excercise 8.1/1 (a)(b)(c)