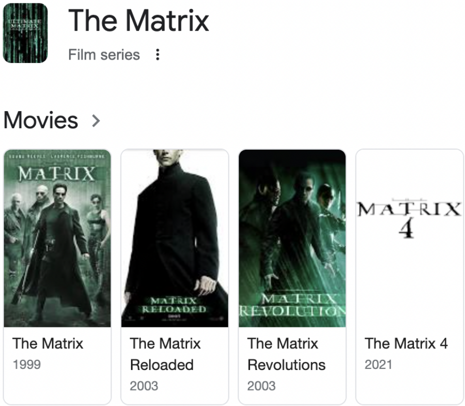
- MA351 Elementary Linear Algebra: basic concepts, intuition and an introduction to matrices, without emphasis on proofs.
- MA353 Linear Algebra II: essentially the same content but in an abstract language, requiring proofs.
- Different sections of MA351: different textbook/homework/exams. For sections under the same professor, the same homework/exams.

What are matrices?

1. The matrix film series:



2. A 2D array of numbers:

$$\begin{pmatrix} 3 & 2 & 1 & 4 \\ 0 & -1 & 9 & 7 \\ 5 & 6 & 0 & 1 \end{pmatrix}$$

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  $\int_a^b F'(x)dx = F(b) - F(a)$.
- Taylor expansion: $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$.
  Every function (curve) can be approximated by a linear function (a line), locally around x_0.
- Multivariable Taylor expansion:

$$f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y$$
$$g(x_0 + \Delta x, y_0 + \Delta y) \approx g(x_0, y_0) + g_x(x_0, y_0)\Delta x + g_y(x_0, y_0)\Delta y$$

  Let $\mathbf{F}(x, y) = \begin{pmatrix} f(x, y) \\ g(x, y) \end{pmatrix}$, then

$$\mathbf{F}(x_0 + \Delta x, y_0 + \Delta y) \approx \mathbf{F}(x_0, y_0) + \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

  Matrices naturally emerge when we approximate any function by a linear one.
  Approximations are common in applications: how does a calculator compute e^x?

Linear system

We can rewrite a system of two equations

$$\begin{cases} 4x - y = 0 \\ x + y = 1 \end{cases}$$

into a matrix-vector form:

$$\begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

- If we have n equations with n unknown variables, then correspondingly there is an $n \times n$ matrix.
- The 2×2 system can be solved by eliminating one variable first.
- In principle, one can still ask computers to solve an $n \times n$ system by eliminating variables, which would be a very slow algorithm,
- To understand any efficient algorithm, one needs to know a lot about matrices.

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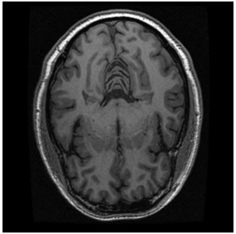
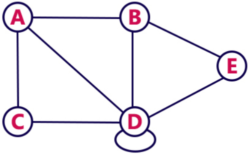
- Pictures are naturally stored as matrices: a 1024×1024 black and white picture is stored as a 1024×1024 matrix. With knowledge of matrices, we can do lots of desired things such as compression of data, which will be explained this semester.



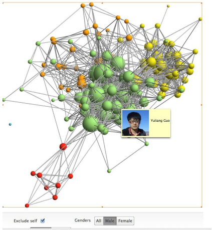
(c) MRI Image

(d) Finger Print

Graph Adjacency Matrix



$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 1 & 0 \\ D & 1 & 1 & 1 & 1 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{array}$$



- A graph with n nodes can be represented as a $n \times n$ matrix A. If there is any edge from node i to node j, then $A_{ij} = 1$. Otherwise $A_{ij} = 0$.
- This matrix is called Graph Adjacency Matrix, which can quantify everything about graph. This is Spectral Graph Theory.
- Example: whether graph is connected (there is a path between any two nodes) is related to eigenvalues of the matrix A.
- Example: in our social media network, is anyone a friend to any other one?

Recommender system

A recommender system, or a recommendation system, is a subclass of information filtering system that seeks to predict the "rating" or "preference" a user would give to an item.

Customers / Users	User1	User2	User3	User4	User5
Items/ Movie Ratings					
Item1	3	5		1	3
Item2	4		2		
Item3		2		5	4

The task is to fill in missing numbers: there are many ways to do it by certain rules. Low-rank matrix fitting is one of them.