Vectors.

- $\mathbb{R}$ is the set of all real numbers.

  $\forall a \in \mathbb{R}$: for any real number a.

  for all ($\forall$)   in / belongs to ($\in$)

- $\mathbb{C}$ is the set of complex numbers.

$\mathbb{R}^n$

$n = 1, 2, 3, \ldots$

- $\mathbb{R}^2 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : \forall x, y \in \mathbb{R} \right\}$

- $\mathbb{R}^3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R} \right\}$

- $\mathbb{R}^{100} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{pmatrix} : \begin{matrix} \forall x_i \in \mathbb{R} \\ 1 \le i \le 100 \end{matrix} \right\}$.

- **Scalars** mean numbers in $\mathbb{R}$ (or $\mathbb{C}$)

Two important basic operations:

① scalar multiplication:

Input: $c \in \mathbb{R}$ $\quad \vec{v} \in \mathbb{R}^n$

$$\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

Output: $c\vec{v} = \begin{pmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{pmatrix} \in \mathbb{R}^n$

Ex:

$$\frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ \pi \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ \frac{\pi}{2} \end{pmatrix}$$

② vector addition.

Input: $\vec{v}, \vec{w} \in \mathbb{R}^n$ $\quad \vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \quad \vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$

Output: $\vec{v} + \vec{w} = \begin{pmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{pmatrix} \in \mathbb{R}^n = \vec{w} + \vec{v}$

associative $\quad (\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w}) = \vec{u} + \vec{v} + \vec{w}$

distribute $\quad c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

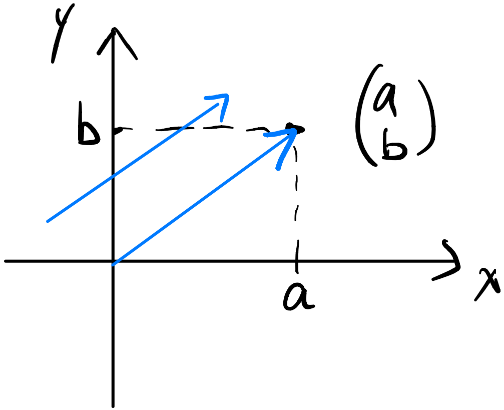
combine these two operations.

linear combination: $c\vec{v} + d\vec{w} = \begin{pmatrix} cv_1 + dw_1 \\ cv_2 + dw_2 \\ \vdots \\ cv_n + dw_n \end{pmatrix}$

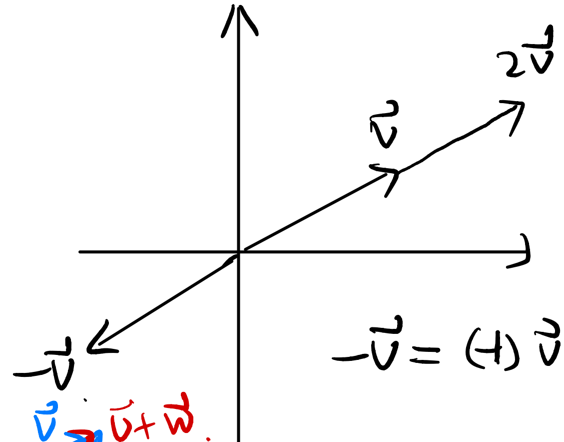
$c, d \in \mathbb{R}$, $\vec{v}, \vec{w} \in \mathbb{R}^n$

$a\vec{u}+b\vec{v}+c\vec{w}$

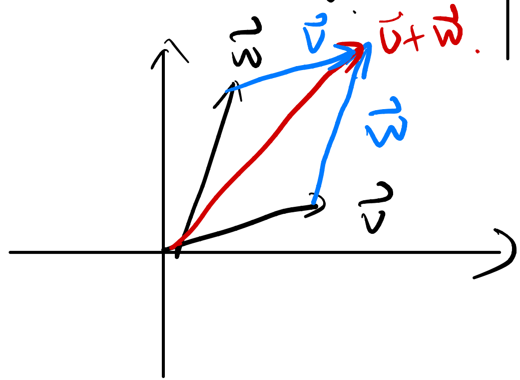
geometric meaning:

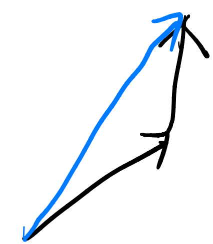


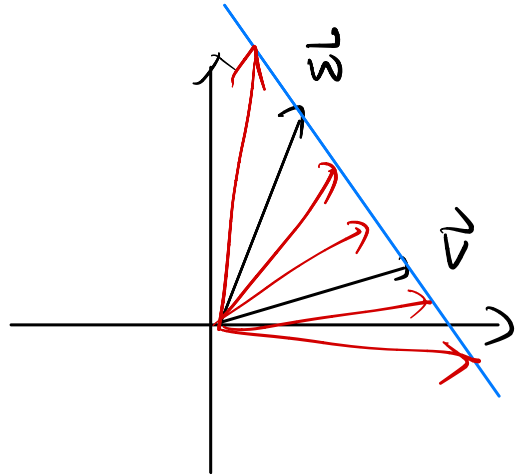
scalar mult.



addition:







$c\vec{v}+d\vec{w}$

$c+d=1$

Q: Fix $\vec{u}\in\mathbb{R}^3$
what is the pic of all vectors $c\vec{u}$, $c\in\mathbb{R}$
Fix $\vec{u},\vec{v}\in\mathbb{R}^3$
— — — — — $c\vec{u}+d\vec{v}$, $c,d\in\mathbb{R}$

Dot product : Input : $\vec{v}, \vec{w} \in \mathbb{R}^n$ $n=3$.

Output : $\vec{v} \cdot \vec{w} \in \mathbb{R}$

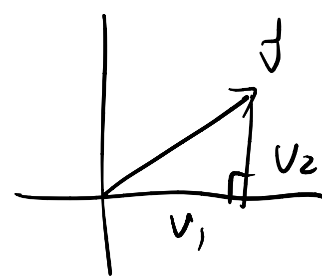
$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \qquad \vec{v}\cdot\vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

- Length $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + v_3^2} \geq 0$

$$\|\vec{v}\| = 0 \iff \vec{v} = \vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\vec{0}$ → zero vector.

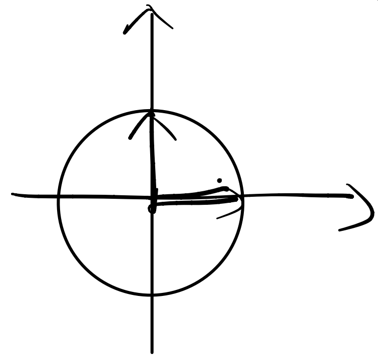
generalize to $\mathbb{R}^n$



$\vec{v}$ $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$

unit vectors: $\vec{v}$ with $\|\vec{v}\| = 1$.

Ex: in $\mathbb{R}^2$ $\quad \vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$$\|c\vec{v}\| = |c|\,\|\vec{v}\|$$