MA 351 Section 041/011
Fall 2023

Homework 6

Due on **Oct 18 before 9am** on gradescope.

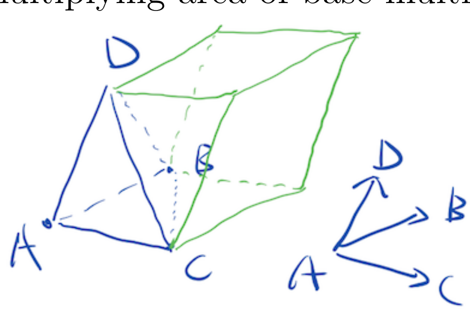
To receive full credit, show necessary reasoning unless it's straightforward computation.

1. (10 pts) Recall that $\vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\vec{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Find all vectors perpendicular to both $\vec{i} + \vec{j}$ and $\vec{i} - \vec{j} + \vec{k}$.

2. (10 pts) Find all vectors in $\mathbb{R}^4$ orthogonal to both $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

3. (20 pts) Given three points A with coordinate $(1, 1, 1)$, B with coordinate $(1, 2, 3)$ and C with coordinate $(2, 2, 2)$. Find the area of the triangle ABC.

4. (20 pts) Given four points with coordinates $A(1, 1, 1)$, $B(1, 2, 3)$, $C(2, 2, 2)$ and $D(1, 1, 2)$. Find the volume of the pyramid $ABCD$ (tetrahedron) by the following method: 1) Find the volume of parallelepiped by taking the absolute value of triple product of three vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$. 2) The volume of the pyramid (tetrahedron) is 1/6 of the volume of the parallelepiped (because the volume formula for tetrahedron is 1/3 multiplying area of base multiplying the height).



5. (20 pts) Compute

$$\begin{vmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{vmatrix}.$$

6. (20 pts) Find projection of the vector $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ onto the row space of the matrix

$$\begin{pmatrix} 1 & 0 & -2 & 3 \\ -1 & 0 & 1 & -2 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{pmatrix}.$$