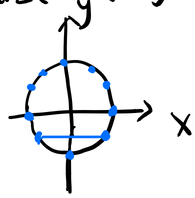
$x^2+y^2+z^2+t^2=1$ defines a 3D manifold

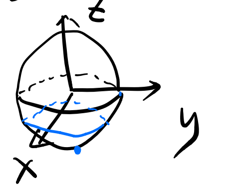
$x^2+y^2+z^2=1$ defines a 2D manifold

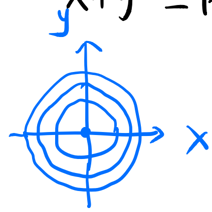
$x^2+y^2=1$ defines a 1D manifold

① Visualize $x^2+y^2=1$: $\underline{x^2=1-y^2}$ defines two end points of an interval for fixed $y\in[-1,1]$

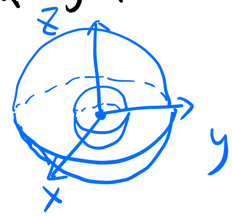


② $x^2+y^2+z^2=1$: $x^2+y^2=1-z^2$ for fixed $z\in[-1,1]$





③ $x^2+y^2+z^2+t^2=1$: $x^2+y^2+z^2=1-t^2$ for fixed $t\in[-1,1]$



$t\in[-1,0]$: the unit ball
$t\in[0,1]$: the unit ball
} 3D

Stokes–Cartan Theorem

$$\int_{\partial\Omega}\omega=\int_{\Omega}d\omega$$

① ω is an $(n-1)$-form
$\partial\Omega$ is the boundary of Ω
(n-1)-dim manifold
$d\omega$ is an n-form
Ω is an n-dimensional manifold

② what d is

③ what the matching orientation is

$$\text{0-form}\xrightarrow{d}\text{1-form}\xrightarrow{d}\text{2-form}\xrightarrow{d}\text{3-form}$$

$$\text{function}\xrightarrow[\nabla f]{\text{grad}}\text{vector field}\xrightarrow[\nabla\times\langle f,g,h\rangle]{\text{Curl}}\text{vector field}\xrightarrow[\nabla\cdot\langle f,g,h\rangle]{\text{Div}}\text{function}$$

$n=2$: Stokes Theorem $\underline{\int_\Omega d\omega = \int_{\partial\Omega} \omega}$

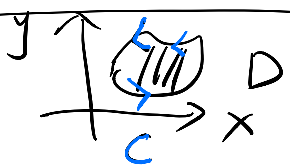
$$\iint_S \underline{\nabla\times\langle f,g,h\rangle\cdot\vec{n}}\,dS = \oint_C \langle f,g,h\rangle\cdot \underbrace{d\vec{s}}_{\langle dx,dy,dz\rangle}$$

Green's Theorem

$$\iint_D \nabla\times\langle P,Q,0\rangle\cdot\langle 0,0,1\rangle\,dxdy = \oint_C Pdx+Qdy$$

$$\iint_D [Q_x - P_y]\,dxdy = \oint_C Pdx+Qdy$$

D is a 2D region



$$\text{Area}(D) = \iint_D 1\,dxdy \underset{\begin{bmatrix}P=0\\Q=x\end{bmatrix}}{=} \oint_C x\,dy$$

$$\underset{\begin{bmatrix}P=-y\\Q=0\end{bmatrix}}{=} \oint_C (-y)\,dx$$

$$\underset{\begin{bmatrix}P=-y/2\\Q=x/2\end{bmatrix}}{=} \underline{\frac{1}{2}\oint_C -ydx+xdy}$$

HW #1: $\underline{\text{Area}(D) = \boxed{\frac{1}{2}\oint_C r^2 d\theta}}$

① General Case $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ Just show $-ydx+xdy = \underline{r^2 d\theta}$

② Do it for a special case for C defined as $\underline{r=\sin\theta}$,

$r=\sin\theta$ show $-ydx+xdy = r^2 d\theta$

HW #2: $\frac{1}{2}\oint_C r^2 d\theta$ for $\underline{r=\sin\theta}$