MA 362 Section 011
Spring 2020

Homework 2

Due on Jan 29th in class.

1. Let $f(x, y) = 3x^2 + y^2$. Sketch the graph of f, as well as some of its level sets and sections.

2. For each of the following functions, find the limit as $(x, y) \to (0, 0)$ using $\varepsilon - \delta$ language, or show that the limit does not exist.

   (a)
   $$\frac{5xy}{2x^2 + 3y^2}$$

   (b)
   $$\frac{x^2y^4e^y \cos x}{x^4 + y^8}$$

   (c)
   $$\frac{x^4y^4e^{x-y}}{2x^4 + y^4}$$

   Hint: If you believe the limit exists, then the inqualities $ab \leq \frac{1}{2}(a^2 + b^2)$ and $ab \leq \frac{1}{4}(a + b)^2$ might help you to get an estimate of these functions. You can also use trivial inequalities like $x^2 + 3y^2 \geq x^2 + y^2$. Then follow what we did in class for dealing with the limit of $\frac{5x^2y^2 \log(x^2+y^2+2)}{2x^2+3y^2}$.

3. Evaluate the partial derivatives of
   $$f(x, y) = \begin{cases} 0, & \text{if } (x, y) = (0, 0), \\ \dfrac{x^2y^3}{x^4 + y^6}, & \text{if } (x, y) \neq (0, 0). \end{cases}$$
   with respect to x and y at $(0, 0)$. Is f differentiable at $(0, 0)$? Explain your answer.

4. Find the equation for the tangent plane at $(1, 2)$ to the graph of $f(x, y) = x^2 + 2y^2 + 1$. Where does this plane intersect the z axis?

5. Find the matrix of partial derivatives of the function $f(x, y) = (xye^{xy}, x \sin y, 5xy^2)$.