MA 362 Section 011
Spring 2020

Homework 4

Due on Feb 12th in class.

1. (20 points) Compute the second-order Taylor formula of the function

$$f(x,y) = e^{x+y^2}\cos(x-y) + \sin(xy) + \sin(x^3+y^3).$$

around the point $(0,0)$.

2. (60 points) For each of the following functions, find all the critical points, and decide if each one is a local maximum, a local minimum, or neither.
   (a) $f(x,y) = x^2 + xy - x + y^2 + 4y + 7$.

   (b) $f(x,y) = x^4 + y^{10}$.

   (c) $f(x,y) = -x^6 - y^8$.

   (d) $f(x,y) = x^4 + y^3$.

   (e) $f(x,y) = x^7y^6$.

3. (20 points) Consider the equation

$$\sin(xyz) + x + y^2 + z^3 = 0.$$

Simply apply
   (a) Is Implicit Function Theorem satisfied to assert that there a differentiable function f such that $x = f(y,z)$ solves the equation near $(0,0,0)$? If so, find $\partial_y f(0,0)$ and $\partial_z f(0,0)$.

   (b) Is Implicit Function Theorem satisfied to assert that there a differentiable function g such that $y = f(x,z)$ solves the equation near $(0,0,0)$? If so, find $\partial_x g(0,0)$ and $\partial_z g(0,0)$.