MA 220 **Final Exam Practice Problems**

The Table of Integrals (pages 502-503 of the text) may be used. The Table of Integrals will be attached to the final exam.

1. If $f(x) = x^2 - 1$, calculate $f(\frac{1}{2})$ and $\frac{1}{f(2)}$.
   A. $f(\frac{1}{2}) = \frac{1}{3}$; $\frac{1}{f(2)} = -\frac{3}{4}$ B. $f(\frac{1}{2}) = -\frac{3}{4}$; $\frac{1}{f(2)} = \frac{1}{3}$ C. $f(\frac{1}{2}) = -\frac{3}{4}$; $\frac{1}{f(2)} = -\frac{4}{3}$
   D. $f(\frac{1}{2}) = 3$; $\frac{1}{f(2)} = \frac{1}{3}$ E. None of these.

2. Find the slope of the line containing the points $(-2, 4)$ and $(6, -3)$.
   A. 4 B. $-\frac{7}{8}$ C. $\frac{1}{4}$ D. $-\frac{8}{7}$ E. $-\frac{1}{2}$

3. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons T and days d is linear, express T as a function of d.
   A. $T = 5d + 280$ B. $T = -44d + 500$ C. $T = 44d + 60$ D. $T = 60d + 44$ E. $T = 60d + 280$

4. When 30 orange trees are planted per acre each tree yields 150 oranges For each additional tree per acre, the yield decreases by 3 oranges per tree. Express the total yield of oranges per acre, Y, as a function of the number of trees planted per acre, x.
   A. $Y = 4500 + 60x - 3x^2$ B. $Y = \frac{1}{3}x + 80$ C. $Y = 150x - 3x^2$ D. $Y = 240x - 3x^2$
   E. $Y = 900 + 3x - 60x^2$

5. The domain of $f(x) = \sqrt{1 - x}$ is:
   A. $(-\infty, 1]$ B. $[1, \infty)$ C. $(-\infty, 1) \cup (1, \infty)$ D. $[-1, 1]$ E. $(-\infty, \infty)$

6. If $f(x) = \sqrt{x + 1}$ and $g(x) = x^2 + 7$ then $(f \circ g)(-1) =$
   A. 0 B. 3 C. $\sqrt{7}$ D. 7 E. $\sqrt{3}$

7. $\lim_{x \to 1} \dfrac{x^2 + 4x - 5}{x^2 - 1} =$
   A. ∞ B. 0 C. 3 D. -3 E. 5

8. $\lim_{x \to \infty} e^{-x} =$
   A. 0 B. 1 C. -1 D. ∞ E. e

9. Find all values where the function $f(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$ is <u>discontinuous</u>.
   A. $x = -2, 1$ B. $x = -2, 3$ C. $x = 3$ D. $x = -2, 1, 3$ E. f is continuous for all values of x.

10. If $f(x) = \dfrac{2}{x}$, find a simplified form for the difference quotient $\dfrac{f(x + h) - f(x)}{h}$.
    A. $-\dfrac{2}{x^2}$ B. $\dfrac{2}{x + h} - \dfrac{2}{x}$ C. $\dfrac{2}{x(x + h)}$ D. $-\dfrac{2}{x(x + h)}$ E. $-\dfrac{2}{x^2 + h}$

11. For what value of a does the function $f(x) = x^2 + ax$ have a relative minimum at $x = 1$?
    A. -2 B. 0 C. 2 D. -1 E. 1

12. The derivative of $\dfrac{x^2 + 1}{x + 5}$ is:
    A. $\dfrac{(x + 5)(2x) - (x^2 + 1)}{(x + 5)^2}$ B. $2x$ C. $\dfrac{(x + 5)(2x)}{(x^2 + 1)^2}$
    D. $\dfrac{(x^2 + 1) + (x + 5)(2x)}{(x + 5)^2}$ E. $\dfrac{(x^2 + 1) - (x + 5)(2x)}{(x + 5)^2}$

13. If $y = (3 - x^2)^3$ then $y'' =$
    A. $-6x(3 - x^2)^2$ B. $24x^2(3 - x^2) - 6(3 - x^2)^2$ C. $6(3 - x^2)$
    D. $24x^2(3 - x^2)$ E. None of these.

14. The line tangent to the graph of $f(x) = x - \dfrac{1}{x}$ at $x = 2$ has slope
    A. 5/4 B. 3/4 C. 3/2 D. 0 E. 1/4

15. A cost function is given by $C(x) = 1000\sqrt{x^2 + 2}$. Use the marginal cost function to estimate the cost of the 11^{th} unit. Round your answer to two decimal places.
    A. 10,099.50 B. 990.15 C. 49.51 D. 99.01 E. 499.15

16. Find all open intervals on which the function $f(x) = 2x^3 - 3x^2 - 12x + 12$ is increasing.
    A. $(-1, 2)$ B. $(-\infty, -1)$ C. $(2, \infty)$ D. $(-\infty, -1)$ and $(2, \infty)$ E. $(-1, 2)$ and $(2, \infty)$

17. If the concentration $C(t)$ of a certain drug remaining in the bloodstream t minutes after it is injected is given by $C(t) = t/(5t^2 + 125)$, then the concentration is a maximum when $t =$
    A. 25 B. 15 C. 5 D. 10 E. There is no maximum.

18. If $f(x) = 2x^4 - 6x^2$ then which one of the following is true?
    A. f has a relative max. at $x = \pm\sqrt{3/2}$ and a relative min at $x = 0$.
    B. f has a relative max. at $x = 0$ and a relative min. at $x = \pm\sqrt{3/2}$.
    C. f has a relative max. at $x = -\sqrt{3/2}$ and a relative min. at $x = \sqrt{3/2}$.
    D. f has no relative max. points, but has relative min. at $x = \pm\sqrt{3/2}$.
    E. None of these.

19. The derivative of a function f is $f'(x) = x^2 - \dfrac{8}{x}$. Then at $x = 2$, f has
    A. an inflection point B. a relative maximum C. a vertical tangent
    D. a vertical asymptote E. a relative minimum

20. If $f(x) = \frac{1}{3}x^3 - 9x + 2$. Then on the closed interval $0 \le x \le 4$,
    A. f has an absolute max. at $x = 3$ and an absolute min. at $x = 0$.
    B. f has an absolute max. at $x = 4$ and an absolute min. at $x = 3$.
    C. f has an absolute max. at $x = 0$ and an absolute min. at $x = 4$.
    D. f has an absolute max. at $x = 0$ and an absolute min. at $x = 3$.
    E. None of these.

21. A cost function is given by $C(x) = 1000\sqrt{x^3 + 1}$. Find the marginal cost when $x = 2$.
    A. \$166.67 B. \$333.33 C. \$4000 D. \$2000 E. \$1000

22. A display case is in the shape of a rectangular box with a square base and open top. Suppose the volume is 21 cubic ft If x is the length of one side of the base, what value should x have to minimize the surface area? Give your answer correct to two decimal places.
    A. 2.78 ft. B. 3.48 ft. C. 4.58 ft. D. 6.48 ft. E. 9.17 ft.

23. A manufacturer determines that in order to sell x units of a product, the price per unit must be $p = 1000 - x$. The manufacturer also determines that the total cost of producing x units is $C(x) = 3000 + 20x$. Calculate the maximum profit.
    A. \$490 B. \$237,100 C. \$121,500 D. \$23,000 E. There is no maximum.

24. If $y = e^{x^2}$ then $\dfrac{dy}{dx} =$

A. e^{x^2} B. $x^2e^{x^2-1}$ C. $2xe^{x^2-1}$ D. $2xe^{x^2}$ E. e^{2x}

25. If $y = \ln(1-x^2)$ then $\dfrac{dy}{dx} =$

A. $\dfrac{1}{1-x^2}$ B. $\dfrac{2x}{\sqrt{1-x^2}}$ C. $\dfrac{-2x}{1-x^2}$ D. $\dfrac{1}{2(1-x^2)}$ E. $\dfrac{2x}{1-x^2}$

26. A population grows exponentially ($Q = Q_0e^{kt}$). In 1960 it was 50,000 and in 1965 it was 100,000. What was the population in 1970?

A. 200,000 B. 150,000 C. 250,000 D. 300,000 E. 225,000

27. Evaluate $\displaystyle\int_0^4 3\sqrt{2x+1}dx$.

A. 27 B. 48 C. 52 D. 26 E. 35

28. Evaluate $\displaystyle\int \left(\frac{2}{x} - \sqrt{x}\right) dx$.

A. $\ln x - 2/\sqrt{x} + C$ B. $-2/x^2 - x^{-1/2}/2 + C$ C. $2\ln x - 2x^{3/2}/3 + C$.
D. $-2/x^2 - 2x^{3/2}/3 + C$ E. $2\ln x - 1/2\sqrt{x} + C$

29. Evaluate $\displaystyle\int \frac{1}{(3x-1)^4}dx$.

A. $-\dfrac{12}{(3x-1)^5} + C$ B. $-\dfrac{1}{9(3x-1)^3} + C$ C. $\dfrac{1}{(3x-1)^3} + C$
D. $-\dfrac{1}{3(3x-1)^3} + C$ E. $-\dfrac{4}{(3x-1)^5} + C$

30. Evaluate $\displaystyle\int e^{3-2x}dx$.

A. $-2e^{3-2x} + C$ B. $-\frac{1}{2}e^{3-2x} + C$ C. $\dfrac{e^{4-2x}}{4-2x}$ D. $\frac{1}{3}e^{3-2x} + C$ E. $\dfrac{e^{3-2x}}{3-2x} + C$

31. Evaluate $\displaystyle\int_1^2 \frac{dx}{3x+1}$. Round your answer to four decimal places.

A. 0.5596 B. 0.6486 C. 1.9459 D. 0.0810 E. 0.1865

32. Evaluate $\displaystyle\int_0^1 x(x^2+1)^5dx$.

A. $\frac{21}{4}$ B. $\frac{16}{3}$ C. $\frac{21}{2}$ D. $\frac{32}{3}$ E. $\frac{24}{3}$

33. The area of the region bounded by the curves $y = x^2 + 1$ and $y = 3x + 5$ is:

A. $\frac{125}{6}$ B. $\frac{56}{3}$ C. $\frac{27}{2}$ D. $\frac{25}{6}$ E. $\frac{32}{3}$

34. Use the table of integrals to evaluate $\displaystyle\int 2x\ln x dx$.

A. $x^2\ln x - x^2/2 + C$ B. $\frac{1}{2}x^2\ln x - \frac{1}{2}x + C$ C. $\frac{1}{2}x^2\ln x - \frac{1}{6}x^3 + C$
D. $x\ln x^2 + 1/x + C$ E. None of these.

35. Use the table of integrals to evaluate $\displaystyle\int \frac{dx}{\sqrt{x^2-9}}$.

A. $\frac{1}{6}\ln|\dfrac{x-3}{x+3}| + C$ B. $\ln|x + \sqrt{x^2-9}| + C$ C. $\ln|x - \sqrt{x^2-9}| + C$ D. $\ln|x - \sqrt{x^2+9}| + C$
E. None of these.

36. Find the average value of $f(x) = x^2$ over the interval $1 \leq x \leq 4$.
A. $\frac{17}{2}$ B. $\frac{15}{2}$ C. 21 D. $\frac{65}{3}$ E. 7

37. Find a function f whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point (2,10).
A. $f(x) = -\frac{1}{3}(5-x^2)^{3/2}$ B. $f(x) = \frac{2}{3}(5-x^2)^{3/2} + \frac{28}{3}$ C. $f(x) = \frac{1}{3}(5-x^2)^{3/2} + \frac{29}{3}$
D. $f(x) = -\frac{1}{3}(5-x^2)^{3/2} + \frac{31}{3}$ E. $f(x) = \frac{3}{2}(5-x^2)^{3/2} + \frac{17}{2}$

38. It is estimated that t years from now the population of a certain town will be increasing at a rate of $5+3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will be population increase over the next 8 years?
A. 100 B. 9,760 C. 6,260 D. 24,760 E. 17,260

39. Evaluate the following improper integral: $\displaystyle\int_0^\infty xe^{-x^2}\,dx$
A. $-\frac{1}{2}$ B. 1 C. $\frac{1}{2}$ D. $\frac{5}{2}$ E. The integral diverges.

40. Find the value of k so that $f(x) = k(3-x)$ is a probability density function on the interval $[0,3]$.
A. $k = \frac{1}{9}$ B. $k = -\frac{2}{3}$ C. $k = -\frac{1}{3}$ D. $k = \frac{2}{9}$ E. $k = \frac{1}{6}$

41. Records indicate that t hours past midnight, the temperature at the West Lafayette airport was $f(t) = -0.3t^2 + 4t + 10$ degrees Celsius. What was the average temperature at the airport between 2:00 A.M. and 7:00 A.M.? Round your answer to the nearest degree.
A. 3° B. 27° C. 21° D. 5° E. 18°

42. Let $f(x)$ be the probability density function defined on the interval $[0,\infty)$ by $f(x) = \dfrac{3}{x^4}$. Calculate $P(x \geq 2)$.
A. 1 B. $\frac{3}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E.$\frac{1}{8}$

43. The probability density function for the life span of light bulbs manufactured by a certain company is $f(t) = 0.01e^{-0.01t}$ where t denotes the life span in hours of a randomly selected bulb, $0 \leq t < \infty$. What is the probability that the life span of a randomly selected bulb is less than or equal to 10 hours? Round your answer to three decimal places.
A. 0.009 B. 0.095 C. 0.905 D. 0.090 E. 0.303

Answers

1. B; 2. B; 3. C; 4. D; 5. A; 6. B; 7. C; 8. A; 9. B; 10. D; 11. A; 12. A; 13. B; 14. A; 15. B; 16. D; 17. C; 18. B; 19. E; 20. D; 21. D; 22. B; 23. B; 24. D; 25. C; 26. A; 27. D; 28. C; 29. B; 30. B; 31. E; 32. A; 33. A; 34. A; 35. B; 36. E; 37. D; 38. B; 39. C; 40. D; 41. C; 42. E; 43. B