MA 222 - FORMULAS

Table of Laplace Transforms

$f(t)$	$F(s)$	$f(t)$	$F(s)$
1. 1	$\frac{1}{s}$	9. $t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
2. t	$\frac{1}{s^2}$	10. $1-\cos at$	$\frac{a^2}{s(s^2+a^2)}$
3. t^n	$\frac{n!}{s^{n+1}}$	11. $at-\sin at$	$\frac{a^3}{s^2(s^2+a^2)}$
4. e^{at}	$\frac{1}{s-a}$	12. $\sin at - at\cos at$	$\frac{2a^3}{(s^2+a^2)^2}$
5. $\sin at$	$\frac{a}{s^2+a^2}$	13. $t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
6. $\cos at$	$\frac{s}{s^2+a^2}$	14. $\sin at + at\cos at$	$\frac{2as^2}{(s^2+a^2)^2}$
7. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	15. $t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
8. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		

Laplace Transforms of Derivatives: $L\{y'\} = sY(s) - y(0), \qquad L\{y''\} = s^2Y(s) - sy(0) - y'(0),$ where $Y(s) = L\{y\}$.

Linear differential equations: $y' + P(x)y = Q(x)$ has solution y, where $ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx + C$

Taylor Series: $f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$

Maclaurin Series: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots$

Examples:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \quad (\text{for all } x), \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \quad (\text{for all } x),$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \quad (-1 < x \le 1), \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \quad (\text{for all } x)$$

Fourier Series: If f is periodic with period $2p$,

$$f(t) = \frac{a_0}{2} + a_1\cos\frac{\pi t}{p} + a_2\cos\frac{2\pi t}{p} + \ldots + a_n\cos\frac{n\pi t}{p} + \ldots$$
$$+ b_1\sin\frac{\pi t}{p} + b_2\sin\frac{2\pi t}{p} + \ldots + b_n\sin\frac{n\pi t}{p} + \ldots$$

where

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(t)dt, \quad a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\frac{n\pi t}{p}\,dt \quad (n \ne 0), \quad b_n = \frac{1}{p}\int_{-p}^{p} f(t)\sin\frac{n\pi t}{p}\,dt$$