MA 224 **Final Exam Practice Problems**

The Table of Integrals (pages 502-503 of the text) and the Formula Page may be used. They will be attached to the final exam.

1. If $f(x, y) = (xy + 1)^2 - \sqrt{y^2 - x^2}$, evaluate $f(-2, 1)$.
   A. 1 B. $1 - \sqrt{5}$ C. Not defined D. $-1 - \sqrt{5}$ E. $-1 - \sqrt{3}$

2. A paint store carries two brands of latex paint. Sales figures indicate that if the first brand is sold for x_1 dollars per gallon and the second for x_2 dollars per gallon, the demand for the first brand will be $D_1(x_1, x_2) = 100 + 5x_1 - 10x_2$ gallons per month and the demand for the second brand will be $D_2(x_1, x_2) = 200 - 10x_1 + 15x_2$ gallons per month. Express the paint store's total monthly revenue, R, as a function of x_1 and x_2.
   A. $R = x_1D_1(x_1, x_2) + x_2D_2(x_1, x_2)$ B. $R = D_1(x_1, x_2) + D_2(x_1, x_2)$
   C. $R = D_1(x_1, x_2)D_2(x_1, x_2)$ D. $R = x_2D(x_1, x_2) + x_1D_2(x_1, x_2)$ E. None of these.

3. Compute $\dfrac{\partial z}{\partial x}$, where $z = \ln(xy)$
   A. $\dfrac{1}{x}$ B. $\dfrac{1}{y}$ C. $\dfrac{1}{xy}$ D. $\dfrac{1}{x} + \dfrac{1}{y}$ E. $\dfrac{y}{x}$

4. Compute f_{uv} if $f = uv + e^{u+2v}$
   A. 0 B. $u + 2e^{u+2v}$ C. $v + 2e^{u+2v}$ D. $1 + 2e^{u+2v}$ E. $1 + e^{u+2v}$

5. Find and classify the critical points of $f(x, y) = (x - 2)^2 + 2y^3 - 6y^2 - 18y + 7$.
   A. (2,3) saddle point; (2,−1) relative minimum
   B. (2,3) relative maximum; (2,−1) relative minimum
   C. (2,3) relative minimum; (2,−1) relative maximum
   D. (2,3) relative maximum; (2,−1) saddle point
   E. (2,3) relative minimum; (2,−1) saddle point

6. A manufacturer sells two brands of foot powder, brand A and brand B. When the price of A is x cents per can and the price of B is y cents per can the manufacturer sells $40 - 8x + 5y$ thousand cans of A and $50 + 9x - 7y$ thousand cans of B. The cost to produce A is 10 cents per can and the cost to produce B is 20 cents per can. Determine the selling price of brand A which will maximize the profit.
   A. 40 cents B. 45 cents C. 15 cents D. 50 cents E. 35 cents

7. Use the total differential to estimate the change in z at (1,3) if $\dfrac{\partial z}{\partial x} = 2x - 4$, $\dfrac{\partial z}{\partial y} = 2y + 7$, the change in x is 0.3 and the change in y is 0.5.
   A. 7.1 B. 2.9 C. 4.9 D. 5.9 E. 6.3

8. Using x worker-hours of skilled labor and y worker-hours of unskilled labor, a manufacturer can produce $f(x, y) = x^2y$ units. Currently 16 worker-hours of skilled labor and 32 worker-hours of unskilled labor are used. If the manufacturer increases the unskilled labor by 10 worker-hours, use calculus to estimate the corresponding change that the manufacturer should make in the level of skilled labor so that the total output will remain the same.
   A. Reduce by 4 hours. B. Reduce by 10 hours. C. Reduce by $\frac{5}{4}$ hours.
   D. Reduce by $\frac{5}{2}$ hours. E. Reduce by 5 hours.

9. Find the maximum value of the function $f(x, y) = 20x^{3/2}y$ subject to the constraint $x+y = 60$. Round your answer to the nearest integer.
   A. 84,654 B. 188,334 C. 4,320 D. 259,200 E. 103,680

10. Evaluate $\int \left(\frac{2}{x} - \sqrt{x}\right) dx$.
   A. $\ln|x| - 2/\sqrt{x} + C$ B. $-2/x^2 - x^{-1/2}/2 + C$ C. $2\ln|x| - 2x^{3/2}/3 + C$
   D. $-2/x^2 - 2x^{3/2}/3 + C$ E. $2\ln|x| - \frac{1}{2\sqrt{x}} + C$

11. Evaluate $\int \frac{1}{(3x-1)^4} dx$.
   A. $-\frac{12}{(3x-1)^5} + C$ B. $-\frac{1}{9(3x-1)^3} + C$ C. $\frac{1}{(3x-1)^3} + C$
   D. $-\frac{1}{3(3x-1)^3} + C$ E. $-\frac{4}{(3x-1)^5} + C$

12. Evaluate $\int e^{3-2x} dx$.
   A. $-2e^{3-2x} + C$ B. $-\frac{1}{2}e^{3-2x} + C$ C. $\frac{e^{4-2x}}{4-2x} + C$ D. $\frac{1}{3}e^{3-2x} + C$ E. $\frac{e^{3-2x}}{3-2x} + C$

13. Find a function f whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point (2,10).
   A. $f(x) = -\frac{1}{3}(5-x^2)^{3/2}$ B. $f(x) = \frac{2}{3}(5-x^2)^{3/2} + \frac{28}{3}$ C. $f(x) = \frac{1}{3}(5-x^2)^{3/2} + \frac{29}{3}$
   D. $f(x) = -\frac{1}{3}(5-x^2)^{3/2} + \frac{31}{3}$ E. $f(x) = \frac{3}{2}(5-x^2)^{3/2} + \frac{17}{2}$

14. Evaluate $\int x\ln(x^2) dx$.
   A. $x^2\ln x - x^2/2 + C$ B.$\frac{1}{2}x^2\ln x^2 - \frac{1}{2}x + C$ C. $\frac{1}{2}x^2\ln x^2 - \frac{1}{6}x^3 + C$
   D. $x\ln x^2 + 1/x + C$ E. None of these.

15. The area of the region bounded by the curves $y = x^2 + 1$ and $y = 3x + 5$ is
   A. $\frac{125}{6}$ B. $\frac{56}{3}$ C. $\frac{27}{2}$ D. $\frac{25}{6}$ E. $\frac{32}{3}$

16. Find the average value of $f(x) = x^2$ over the interval $1 \leq x \leq 4$.
   A. $\frac{17}{2}$ B. $\frac{15}{2}$ C. 21 D. $\frac{65}{3}$ E. 7

17. A calculator manufacturer expects that x months from now consumers will be buying 1000 calculators a month at a price of $20 + 3\sqrt{x}$ dollars per calculator. What is the total revenue the manufacturer can expect from the sale of calculators over the next 4 months?
   A. \$8,000 B. \$16,000 C. \$96,000 D. \$192,000 E. None of these.

18. The general solution of the differential equation $\frac{dy}{dx} = 2y + 1$ is
   A. $x = y^2 + y + C$ B. $2y + 1 = Ce^{2x}$ C. $y = 2xy + x + C$ D. $y = Ce^{2x} - 2y - 1$
   E. None of these.

19. The value, V, of a certain \$1500 IRA account grows at a rate equal to 13.5% of its value. Its value after t years is
   A. $V = 1500e^{-0.135t}$ B. $V = 1500 + 0.135t$ C. $V = 1500e^{0.135t}$ D. $V = 1500(1 + 0.135t)$
   E. $V = 1500\ln(0.135t)$

20. Evaluate $\int \frac{dx}{\sqrt{9x^2-4}}$.

    A. $\ln|3x+\sqrt{9x^2-4}|+C$ B. $\frac{1}{3}\ln|x+\sqrt{x^2-4}|+C$ C. $\frac{1}{3}\ln|x+\sqrt{x^2+(4/9)}|+C$
    D. $\frac{1}{3}\ln|x+\sqrt{x^2-(4/9)}|+C$ E. $\ln|x+\sqrt{x^2-(4/9)}|+C$.

21. Evaluate $\int_1^2 x\ln(x^2)dx$. Round your answer to two decimal places.

    A. 1.27 B. -0.30 C. 2.27 D. 0.77 E. 1.77

22. It is estimated that t years from now the population of a certain town will be increasing at a rate of $5+3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will be population increase over the next 8 years?

    A. 100 B. 9,760 C. 6,260 D. 24,760 E. 17,260

23. The probability density function for the life span of light bulbs manufactured by a certain company is $f(x) = 0.01e^{-0.01x}$ where x denotes the life span in hours of a randomly selected bulb. What is the probability that the life span of a randomly selected bulb is less than or equal to 10 hours? Round your answer to three decimal places.

    A. 0.009 B. 0.095 C. 0.905 D. 0.090 E. 0.303

24. Calculate the following improper integral $\int_0^\infty xe^{-x^2}dx$

    A. $-\frac{1}{2}$ B. 1 C. $\frac{1}{2}$ D. $\frac{5}{2}$ E. The integral diverges.

25. An object moves so that its velocity after t minutes is given by the formula $v = 20e^{-0.01t}$. The distance it travels during the 10th minute is

    A. $\int_0^{10} 20e^{-0.01t}dt$ B. $\int_9^{10} (-20e^{-0.01t})dt$ C. $\int_0^{10} (-20e^{-0.01t})dt$
    D. $\int_9^{10} 20e^{-0.01t}dt$ E. None of these.

26. Find the value of k such that $f(x) = k(3-x)$ is a probability density function on the interval $[0,3]$.

    A. $k=\frac{1}{9}$ B. $k=-\frac{2}{3}$ C. $k=-\frac{1}{3}$ D. $k=\frac{2}{9}$ E. $k=\frac{1}{6}$

27. Records indicate that t hours past midnight, the temperature at the West Lafayette airport was $f(t) = -0.3t^2+4t+10$ degrees Celsius. What was the average temperature at the airport between 2:00 A.M. and 7:00 A.M.? Round your answer to the nearest degree.

    A. 3° B. 27° C. 21° D. 5° E. 18°

28. Let $f(x)$ be the probability density function on the interval $[1,\infty)$ defined by $f(x) = \frac{3}{x^4}$. Calculate $P(x \geq 2)$.

    A. 1 B. $\frac{3}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. $\frac{1}{8}$

29. Approximate $\int_0^1 e^{x^2}dx$ using the trapezoidal rule with $n = 4$. Round your answer to two decimal places.

    A. 1.49 B. 2.98 C. 5.96 D. 1.73 E. 1.96

30. The slope of the least-squares line for the points (1,2), (2,4), (4,4), (5,2) is
    A. 0 B. 1 C. 2 D. 3 E. 4

31. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^n$
    A. $\frac{2}{5}$ B. $-\frac{2}{5}$ C. $\frac{3}{2}$ D. $-\frac{3}{2}$ E. The series diverges.

32. Use a Taylor polynomial of degree 2 to approximate $\sqrt{3.8}$. Round your answer to five decimal places.
    A. 1.94936 B. 1.94938 C. 1.94940 D. 1.94947 E. 1.95000

33. Use a Taylor polynomial of degree 2 to approximate $\displaystyle\int_0^{0.1}\frac{100}{x^2+1}\,dx$. Round your answer to five decimal places.
    A. 9.96687 B. 10.00000 C. 9.96677 D. 9.66667 E. 9.96667

34. Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{n3^nx^n}{5^{n+1}}$.
    A. 5/3 B. 1 C. 3/25 D. 3/5 E. ∞

35. Find the Taylor series of $f(x) = \dfrac{x}{2+x^2}$ at $x = 0$.
    A. $\displaystyle\sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+1}}$ B. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^nx^{2n+1}}{2^n}$ C. $\displaystyle\sum_{n=0}^{\infty}(-1)^n2^{n-1}x^{2n+1}$ D. $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{2^{n-1}}$ E. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^nx^{2n+1}}{2^{n+1}}$

Answers

1. C; 2. A; 3. A; 4. D; 5. E; 6. A; 7. D; 8. D; 9. E; 10. C; 11. B; 12. B; 13. D; 14. A;
15. A; 16. E; 17. C; 18. B; 19. C; 20. D; 21. A; 22. B; 23. B; 24. C; 25. D; 26. D; 27. C;
28. E; 29. A; 30. A; 31. B; 32. B; 33. E; 34. A; 35. E