

MA 15400

Fall 2012

Exam 1

Solutions

PYTHAGOREAN IDENTITIES		
$\sin^2 \theta + \cos^2 \theta = 1$	$1 + \tan^2 \theta = \sec^2 \theta$	$1 + \cot^2 \theta = \csc^2 \theta$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

1. Approximate $\sec(12.4^\circ)$ to four decimal places.

A. 3.5003

B. 0.0014

C. 4.6569

D. 1.0239

E. None of the above

$$= \frac{1}{\cos(12.4^\circ)} = \frac{1}{0.9767} = 1.0239$$

2. Find the area of the sector of the circle with radius 8.2 cm, whose central angle θ subtends an arc of 12.6 cm. Round your answer to one decimal place.A. 51.7 cm^2 B. 38.5 cm^2 C. 62.5 cm^2 D. 42.1 cm^2

E. None of the above.

$$S = r\theta$$

$$12.6 = 8.2\theta$$

$$\theta = \frac{12.6}{8.2}$$

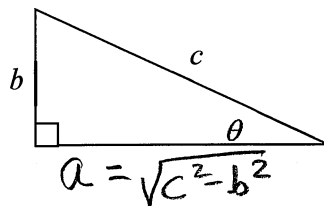
$$\theta = 1.5366$$

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(8.2)^2(1.5366)$$

$$A = 51.6600$$

3. Which statement is true for the given right triangle?

A. $\cos \theta = \frac{b}{c}$ B. $\tan \theta = \frac{b}{\sqrt{c^2 - b^2}}$ C. $\sin \theta = \frac{\sqrt{c^2 - b^2}}{b}$ D. $\sec \theta = \frac{\sqrt{c^2 - b^2}}{c}$ E. $\cot \theta = \frac{c}{b}$ 

$$c^2 = a^2 + b^2$$

$$c^2 - b^2 = a^2$$

$$\sqrt{c^2 - b^2} = a$$

$$\sin \theta = \frac{b}{c}$$

$$\cos \theta = \frac{\sqrt{c^2 - b^2}}{c}$$

$$\tan \theta = \frac{b}{\sqrt{c^2 - b^2}}$$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

4. **Stonehenge blocks:** Stonehenge in Salisbury Plains, England, was constructed using solid stone blocks weighing over 97000 pounds each. Lifting a single stone required 550 people, who pulled the stone up a ramp inclined at an angle of 8° . To the nearest foot, approximate the distance that a stone was moved **along the ramp** in order to raise it to a height of 34 feet above the level ground.

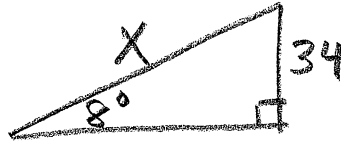
A. 244 feet

B. 402 feet

C. 254 feet

D. 411 feet

E. None of the above.



$$\sin 8^\circ = \frac{34}{X}$$

$$X = \frac{34}{\sin 8^\circ}$$

$$X = 244.3$$

5. Express the angle θ to the nearest ten-thousandths of a degree.

A. $\theta = 274.5850^\circ$ B. $\theta = 274.9833^\circ$ C. $\theta = 274.8850^\circ$ D. $\theta = 274.6833^\circ$

E. None of the above.

$$\theta = 274^\circ 53' 6''$$

Method #1

$$\frac{53'}{60} = 0.8833$$

$$\frac{6''}{3600''} = +0.0017$$

$$274.8850^\circ$$

Method #2

$$\frac{\frac{6}{60} + 53}{60} = 0.8850$$

$$274.8850^\circ$$

6. $\cos(-\alpha)\csc(-\alpha)$ is equivalent to which of the following?

A. $\cot \alpha$ B. $-\cot \alpha$ C. $\tan \alpha$ D. $-\tan \alpha$

E. None of the above

$$\left(\frac{\cos \alpha}{1} \right) \left(\frac{1}{-\sin \alpha} \right) = -\frac{\cos \alpha}{\sin \alpha}$$

$$= -\cot \alpha$$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

7. Find the exact value of $\tan \theta$ if θ is in standard position and the terminal side of θ is in quadrant II and is parallel to the line $5x + 4y = 8$.

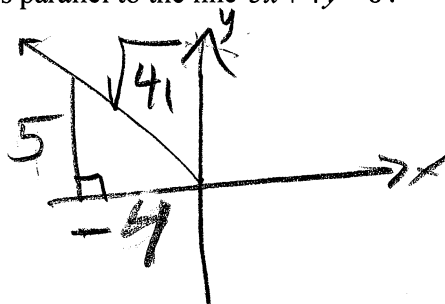
A. $\frac{-4}{\sqrt{41}}$

B. $\frac{5}{4}$

C. $\frac{5}{\sqrt{41}}$

D. $\frac{-5}{4}$

E. None of the above.



$$5x + 4y = 8$$

$$4y = -5x + 8$$

$$y = -\frac{5}{4}x + 2$$

$$m = -\frac{5}{4}$$

$$\therefore \tan \theta = -\frac{5}{4}$$

$$\tan \theta = \frac{y}{x} = -\frac{5}{4}$$

$$c^2 = 4^2 + 5^2$$

$$c^2 = 16 + 25 = 41$$

$$c = \sqrt{41}$$

8. If $\sec \theta = 4$ and $\cot \theta < 0$, find the exact value of $\sin \theta$.

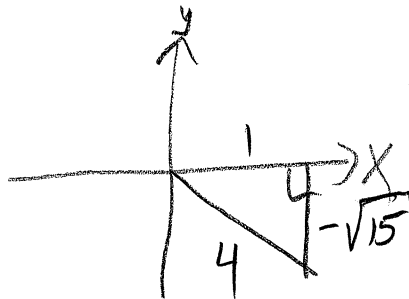
A. $\frac{\sqrt{17}}{8}$

B. $\frac{-1}{4}$

C. $\frac{-\sqrt{17}}{8}$

D. $\frac{1}{4}$

E. None of the above.



$$\sec \theta > 0$$

$$\therefore \cos \theta > 0 \text{ QI, IV}$$

$$\cot \theta < 0$$

$$\therefore \tan \theta < 0, \text{ QII, III}$$

$$\sin \theta = \frac{-\sqrt{15}}{4}$$

$$\sec \theta = 4$$

$$\cos \theta = \frac{1}{4}$$

$$4^2 = 1^2 + b^2$$

$$b = \pm \sqrt{15}$$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

9. Let $P(t) = \left(\frac{-7}{25}, \frac{-24}{25}\right)$ be the point on the unit circle that corresponds to t . Find the exact value of $P(-t + \pi)$.

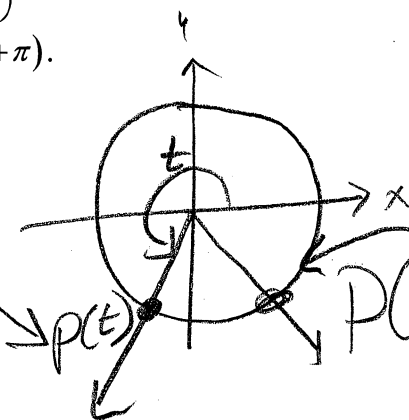
A. $\left(\frac{-7}{25}, \frac{-24}{25}\right)$

B. $\left(\frac{7}{25}, \frac{-24}{25}\right)$

C. $\left(\frac{-7}{25}, \frac{24}{25}\right)$

D. $\left(\frac{7}{25}, \frac{24}{25}\right)$

E. None of the above.



$$P(-t) = \left(\frac{-7}{25}, \frac{24}{25}\right)$$

$$P(-t \pm \pi) = \left(\frac{7}{25}, \frac{-24}{25}\right)$$

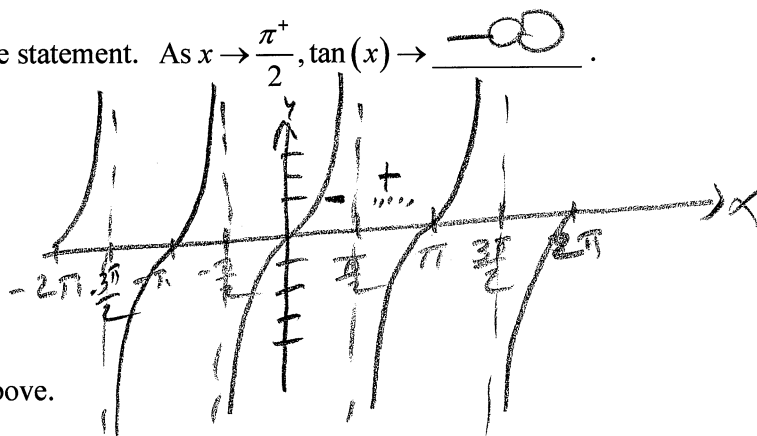
10. Complete the statement. As $x \rightarrow \frac{\pi^+}{2}$, $\tan(x) \rightarrow$ $-\infty$.

A. 1

B. ∞ C. $-\infty$

D. undefined

E. None of the above.



11. $\sin^2 \theta (\csc^2 \theta + \sec^2 \theta)$ is equivalent to which of the following?

A. $\sec^2 \theta$

B. $\csc^2 \theta$

C. 1

D. $\cot^2 \theta$

E. $\tan^2 \theta$

$$\sin^2 \theta \left(\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \right) =$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} =$$

$$1 + \tan^2 \theta =$$

$$\sec^2 \theta = \sec^2 \theta$$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

12. What is the y-intercept for the graph of $y = 8 \sin\left(2x + \frac{3\pi}{2}\right)$?

A. $(0, -16)$ B. $(0, 8)$ C. $(0, 16)$ D. $(0, -8)$

E. None of the above.

$$\text{Let } x = 0$$

$$y = 8 \sin\left(0 + \frac{3\pi}{2}\right)$$

$$y = 8(-1)$$

$$y = -8$$

$$(0, -8)$$

13. Find the equation of the graph in the form $y = a \sin(bx + c)$, with $a > 0$, $b > 0$ and least positive real c .

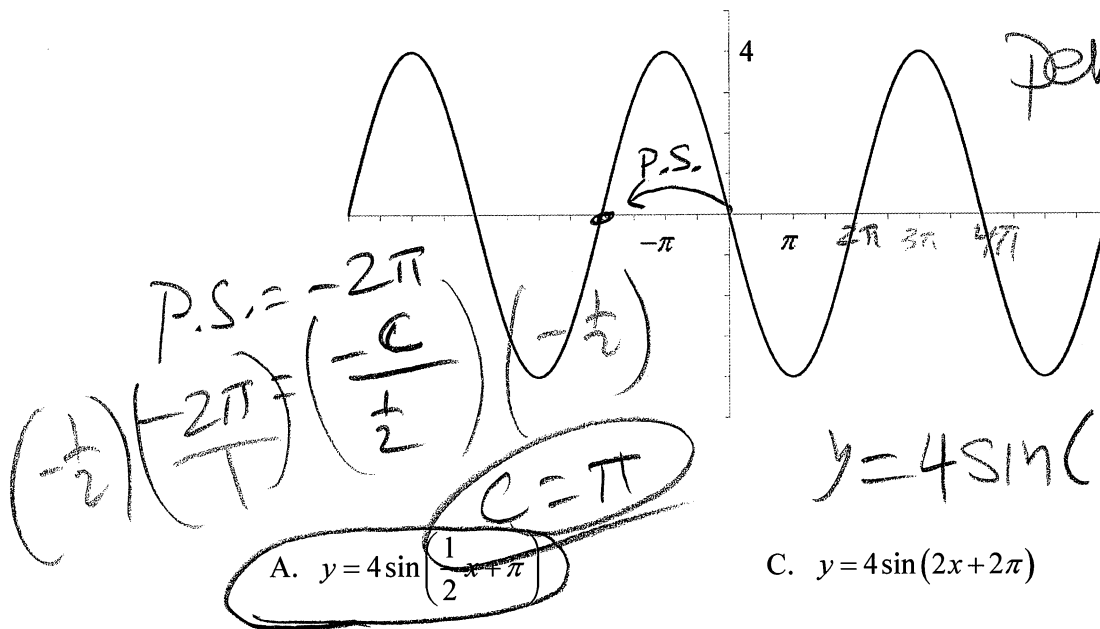
$$\text{Amp} = 4 \quad (a = 4)$$

$$\text{Per} = 4\pi = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$(b = \frac{1}{2})$$

$$y = 4 \sin\left(\frac{1}{2}x + \pi\right)$$



$$\text{A. } y = 4 \sin\left(\frac{1}{2}x + \pi\right)$$

$$\text{C. } y = 4 \sin(2x + 2\pi)$$

$$\text{B. } y = 4 \sin\left(\frac{1}{2}x + 2\pi\right)$$

$$\text{D. } y = 4 \sin(2x + \pi)$$

Covers Lessons 1 to 11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

14. Approximate, to the nearest 0.1° , all angles θ in the interval $[0^\circ, 360^\circ)$ that satisfy the equation $\cos(\theta) = 0.3356$

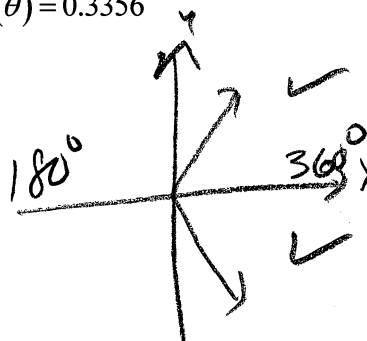
A. $70.4^\circ, 289.6^\circ$

B. $109.6^\circ, 250.4^\circ$

C. $250.4^\circ, 289.6^\circ$

D. $70.4^\circ, 109.6^\circ$

E. None of the above.



$$\theta = \cos^{-1}(0.3356)$$

$$\theta_1 = 70.4^\circ$$

$$\theta_2 = 70.4^\circ$$

$$\theta_2 = 360^\circ - 70.4^\circ$$

$$\theta_2 = 289.6^\circ$$

15. Approximate, to the nearest 0.01 **radians**, all angles θ in the interval $[0, 2\pi)$ that satisfy the equation $\cot(\theta) = -1.7401$

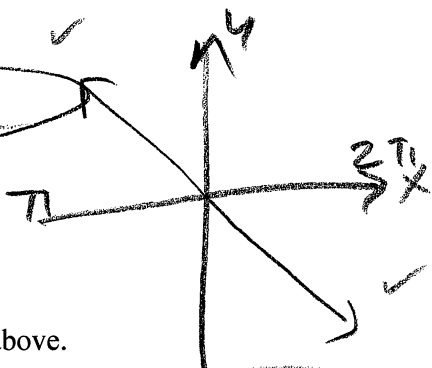
A. 0.35, 3.49

B. 2.62, 5.76

C. 0.52, 3.66

D. 2.79, 5.93

E. None of the above.



$$\cot \theta = -1.7401$$

$$\tan \theta = -0.5747$$

$$\theta = \tan^{-1}(-0.5747)$$

$$\theta = -0.5216$$

$$\theta_2 = 0.5216$$

$$\theta_1 = \pi - 0.5216 = 2.62$$

$$\theta_2 = 2\pi - 0.5216 = 5.76$$

Exam 1 Answers

	Answers
1.	D
2.	A
3.	B
4.	A
5.	C
6.	B
7.	D
8.	E
9.	B
10.	C
11.	A
12.	D
13.	A
14.	A
15.	B