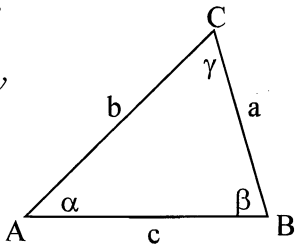


# MA 15400

## Fall 2012

### Exam 2

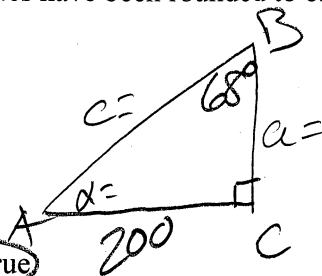
$\sin(u+v) = \sin u \cos v + \cos u \sin v$		$\sin(u-v) = \sin u \cos v - \cos u \sin v$
$\cos(u+v) = \cos u \cos v - \sin u \sin v$		$\cos(u-v) = \cos u \cos v + \sin u \sin v$
$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$		$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$
$\sin(2u) = 2 \sin u \cos u$		$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\sin^2 \theta + \cos^2 \theta = 1$	$\cos(2u) = \cos^2 u - \sin^2 u$	$1 + \cot^2 \theta = \csc^2 \theta$
	$1 + \tan^2 \theta = \sec^2 \theta$	

Covers Lesson 12 to 22, Sections 6.7, 7.2, 7.3, 7.4, and 7.6 (questions 1 to 34)

1. Given triangle  $ABC$ , with  $\gamma = 90^\circ$ ,  $\beta = 68^\circ$ , and  $b = 200.0$ , which one of the following statements is true? Note that all values have been rounded to one decimal place.

A. side  $a = 279.8$ B. side  $c = 533.9$ C. side  $a = 495.0$ D. side  $c = 264.8$ 

E. None of the above statements are true



$$\tan 68^\circ = \frac{200}{a}$$

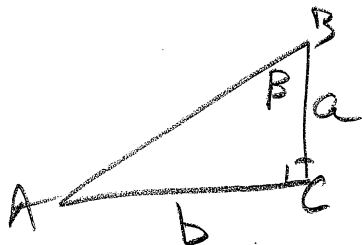
$$a = \frac{200}{\tan 68^\circ}$$

$$\sin 68^\circ = \frac{200}{c}$$

$$c = \frac{200}{\sin 68^\circ} = 215.7$$

$$\begin{aligned} a &= 80.8 \\ c &= 215.7 \\ \alpha &= 22^\circ \end{aligned}$$

2. Given triangle  $ABC$ , with  $\gamma = 90^\circ$ , express side  $b$  in terms of angle  $\beta$  and side  $a$ .

A.  $b = a \cos \beta$ B.  $b = a \tan \beta$ C.  $b = a \sin \beta$ D.  $b = a \cot \beta$ E.  $b = a \sec \beta$ 

$$\tan \beta = \frac{b}{a}$$

$$b = a \tan \beta$$

If solving for  $a$ :

$$\tan \beta = \frac{b}{a}$$

$$a = \frac{b}{\tan \beta} = b \cot \beta$$

3. Express as a trigonometric function of one angle.

$$\sin(46^\circ) \cos(21^\circ) + \cos(46^\circ) \sin(21^\circ)$$

A.  $\cos(67^\circ)$ B.  $\sin(25^\circ)$ C.  $\cos(25^\circ)$ D.  $\sin(67^\circ)$ 

$$\sin(46^\circ + 21^\circ)$$

$$\sin(67^\circ)$$

Covers Lesson 12 to 22, Sections 6.7, 7.2, 7.3, 7.4, and 7.6 (questions 1 to 34)

4. A drawbridge is  $l = 250$  feet long when stretched across a river. As shown in the figure, the two sections of the bridge can be rotated upward through an angle of  $\alpha = 40^\circ$ . **Approximately how far apart are the ends of the two sections** when the bridge is fully opened? Round your answer to one decimal place.

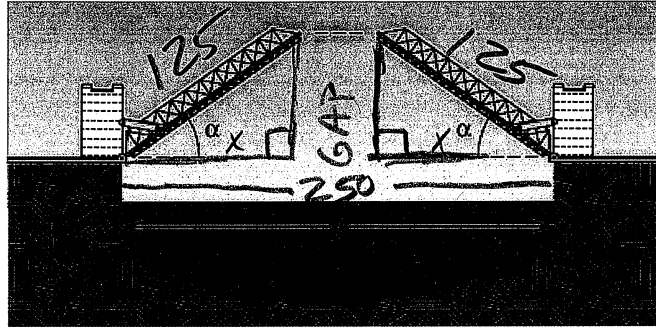
A. 89.3 ft.

B. 80.3 ft.

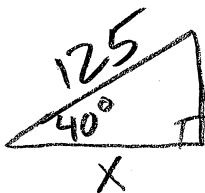
C. 58.5 ft.

D. 55.5 ft.

E. None of the above



$$\frac{250}{2} = 125$$



$$\cos 40^\circ = \frac{x}{125}$$

$$x = 125 \cos 40^\circ$$

$$x = 95.76$$

$$\text{GAP} = 250 - 2x$$

$$= 250 - 2(95.76)$$

$$= 58.5 \text{ ft}$$

5. A builder wishes to construct a ramp 27 feet long that rises to a height of 4.3 feet above level ground. To the nearest tenth of a degree, approximate the angle that the ramp should make with the horizontal.

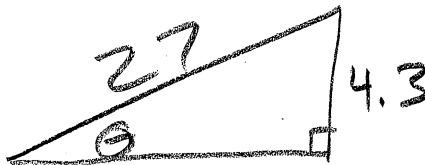
A. 9.2°

B. 10.4°

C. 10.8°

D. 9.7°

E. None of the above



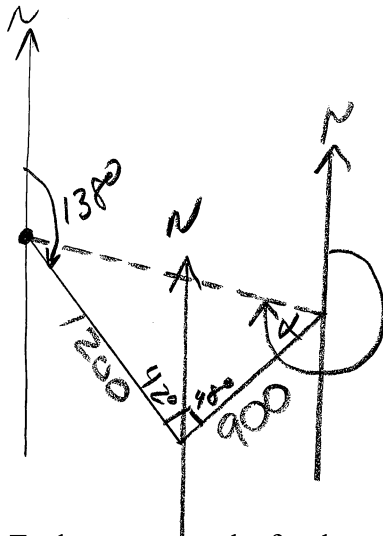
$$\sin \theta = \frac{4.3}{27}$$

$$\theta = \sin^{-1}\left(\frac{4.3}{27}\right)$$

$$\theta = 9.2^\circ$$

Questions 6 and 7:

An airplane, flying at a speed of 600 miles per hour, flies from a point  $A$  in the direction  $138^\circ$  for 2 hour and then flies in the direction  $48^\circ$  for 1.5 hours.



$$D = rt$$

$$D_1 = 600(2) = 1200$$

$$D_2 = 600(1.5) = 900$$

$$180^\circ$$

$$-138^\circ$$

$$\hline 42^\circ$$

$$42^\circ + 48^\circ = 90^\circ$$

6. To the nearest tenth of an hour, how long will it take to get back to point  $A$ ?

A. 1.9 hours

B. 2.2 hours

C. 2.5 hours

D. 2.7 hours

E. None of the above

$$D = rt$$

$$t = \frac{D}{r}$$

$$D^2 = 1200^2 + 900^2$$

$$D = 1500$$

$$t = \frac{1500}{600}$$

$$t = 2.5$$

7. To the nearest degree, in what direction does the plane need to fly in order to get back to point  $A$ ?

A.  $296^\circ$ B.  $255^\circ$ C.  $312^\circ$ D.  $281^\circ$ 

E. None of the above

$180^\circ$  from N to S

$48^\circ$  Alt. Int Angle

$$\tan \alpha = \frac{1200}{900}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\alpha = 53^\circ$$

Ans:

$$180^\circ$$

$$+ 48^\circ$$

$$+ 53^\circ$$

$$\hline 281^\circ$$

Covers Lesson 12 to 22, Sections 6.7, 7.2, 7.3, 7.4, and 7.6 (questions 1 to 34)

8. Find all solutions of the equation using  $n$  as an arbitrary integer.

$$4 \sec \beta = 8$$

A.  $\beta = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

B.  $\beta = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$

C.  $\beta = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n$

D.  $\beta = \frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$

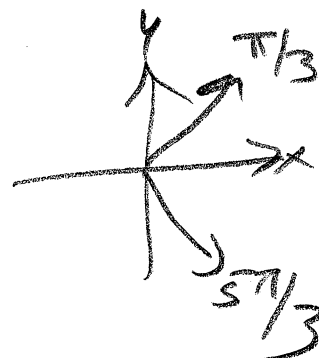
E.  $\beta = \frac{\pi}{4} + 2\pi n, \frac{5\pi}{4} + 2\pi n$

$$\sec \beta = 2$$

$$\cos \beta = \frac{1}{2}$$

$$\beta = \frac{\pi}{3} + 2\pi n$$

$$\beta = \frac{5\pi}{3} + 2\pi n$$



9. Find all solutions of the equation in the interval  $[0, 2\pi)$ .

$$\tan\left(2\theta - \frac{5\pi}{6}\right) = \sqrt{3}$$

A.  $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

B.  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

C.  $\theta = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

D.  $\theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

E. None of the above

n	$\theta$
0	$\frac{7\pi}{12}$
1	$\frac{13\pi}{12}$
2	$\frac{19\pi}{12}$
3	Too big

n	$\theta$
-1	$\frac{\pi}{12}$
-2	Too small

$$2\theta - \frac{5\pi}{6} = \frac{\pi}{3} + \pi n$$

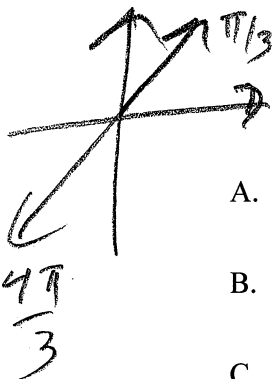
$$2\theta = \frac{5\pi}{6} + \frac{2\pi}{6} + \pi n$$

$$2\theta = \frac{7\pi}{6} + \pi n$$

$$\theta = \frac{7\pi}{12} + \frac{\pi}{2} n$$

Rewrite with common Den.

$$\theta = \frac{7\pi}{12} + \frac{6\pi}{12} n$$



Covers Lesson 12 to 22, Sections 6.7, 7.2, 7.3, 7.4, and 7.6 (questions 1 to 34)

10. From a point 45 meters above level ground, a hiker measures the angle of **depression** to the friend on the ground to be  $57^\circ$ . If the hiker drops her backpack straight down to the ground, approximate the distance her friend will have to walk, along the level ground, to retrieve the backpack. Round to one decimal place.

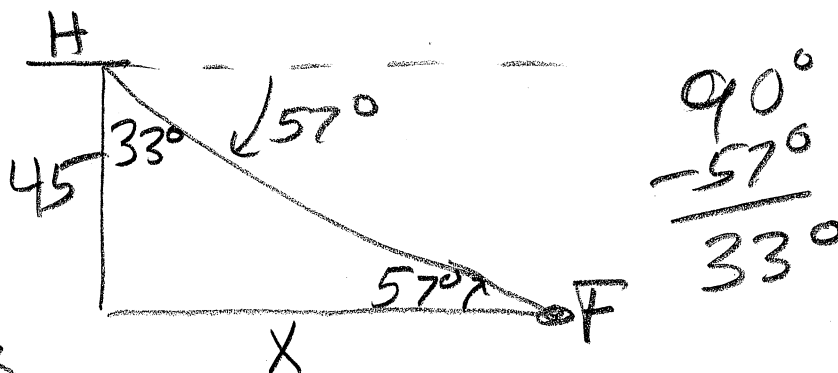
A. 37.7 meters

B. 29.2 meters

C. 69.3 meters

D. 24.5 meters

E. None of the above



$$\tan 33^\circ = \frac{X}{45}$$

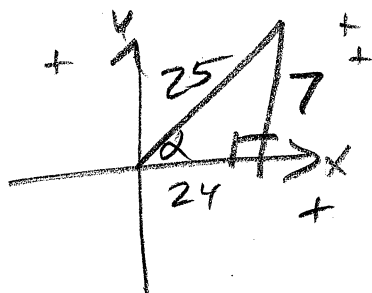
$$45 \tan 33^\circ = X$$

$$X = 29.2$$

11. If  $\cos \alpha = \frac{24}{25}$  and  $\sin \alpha > 0$ , find the exact value of  $\cos\left(\alpha + \frac{\pi}{6}\right)$

A.  $\frac{24\sqrt{3} - 7}{50}$ B.  $\frac{-24\sqrt{3} - 7}{50}$ C.  $\frac{-24\sqrt{3} + 7}{50}$ D.  $\frac{24\sqrt{3} + 7}{50}$ 

E. None of the above



$$25^2 = 24^2 + b^2$$

$$b = 7$$

$$\cos\left(\alpha + \frac{\pi}{6}\right) = \cos \alpha \cos \frac{\pi}{6} - \sin \alpha \sin \frac{\pi}{6}$$

$$= \left(\frac{24}{25}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{7}{25}\right)\left(\frac{1}{2}\right)$$

$$= \frac{24\sqrt{3}}{50} - \frac{7}{50} = \frac{24\sqrt{3} - 7}{50}$$

12. If  $\sec \beta = \frac{11}{6}$ , for  $0^\circ < \beta < 90^\circ$ , find  $\sin(2\beta)$ .

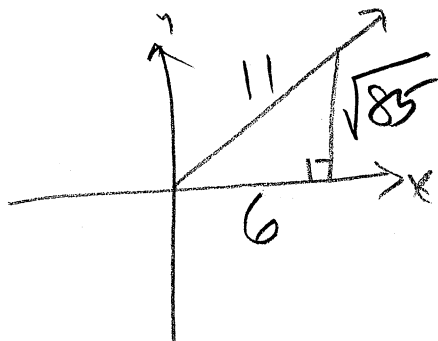
A.  $\frac{12\sqrt{157}}{121}$

B.  $\frac{-12\sqrt{85}}{121}$

C.  $\frac{12\sqrt{85}}{121}$

D.  $\frac{-12\sqrt{157}}{121}$

E None of the above



$$\sec \beta = \frac{11}{6}$$

$$\cos \beta = \frac{6}{11}$$

$$11^2 = 6^2 + b^2$$

$$b = \pm \sqrt{85}$$

$$\sin 2\beta = 2 \sin \beta \cos \beta$$

$$= 2 \left( \frac{\sqrt{85}}{11} \right) \left( \frac{6}{11} \right)$$

$$= \frac{12\sqrt{85}}{121}$$

13. Find the solutions of the equation that are in the interval  $[0, 2\pi)$ .

$$\cos(t) - \sin(2t) = 0$$

A.  $t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

B.  $t = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$

C.  $t = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

D.  $t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$

E None of the above

$$\cos t - 2 \sin t \cos t = 0$$

$$\cos t (1 - 2 \sin t) = 0$$

$$\cos t = 0 \quad 1 - 2 \sin t = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin t = \frac{1}{2}$$

$$t = \frac{\pi}{6}, \frac{5\pi}{6}$$

14. Find the exact value of the expression whenever it is defined.

$$\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) =$$

A.  $\frac{\pi}{3}$

B.  $\frac{2\pi}{3}$

C.  $\frac{-\pi}{3}$

D.  $\frac{5\pi}{3}$

E None of the above

Not on  
SP '14  
Ex. 2

15. Write the expression as an algebraic expression in  $x$  for  $x > 0$ .

$$\sin\left(2\sin^{-1}\left(\frac{x}{4}\right)\right)$$

A.  $\frac{x}{2}$

B.  $\frac{x-x^2}{2}$

C.  $\frac{4x-x^2}{8}$

D.  $\frac{x\sqrt{16-x^2}}{8}$

Not on  
SP '14  
Ex. 2



## Exam 2 Answers

	Answers
1.	E
2.	B
3.	D
4.	C
5.	A
6.	C
7.	D
8.	A
9.	D
10.	B
11.	A
12.	C
13.	A
14.	A
15.	D