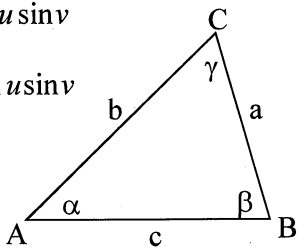


# MA 15400

## Spring 2012

### Exam 2

|   |   |   |                                     |
|---|---|---|-------------------------------------|
| $\sin(u + v) = \sin u \cos v + \cos u \sin v$             |  | $\sin(u - v) = \sin u \cos v - \cos u \sin v$             |                                     |
| $\cos(u + v) = \cos u \cos v - \sin u \sin v$             |   | $\cos(u - v) = \cos u \cos v + \sin u \sin v$             |                                     |
| $\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ |   | $\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$ |                                     |
| $\sin(2u) = 2 \sin u \cos u$                              |   | $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$                |                                     |
| $\sin^2 \theta + \cos^2 \theta = 1$                       | $\cos(2u) = \cos^2 u - \sin^2 u$  |   | $1 + \cot^2 \theta = \csc^2 \theta$ |
|   | $1 + \tan^2 \theta = \sec^2 \theta$   |   |                                     |

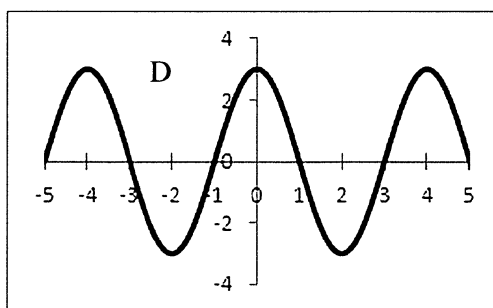
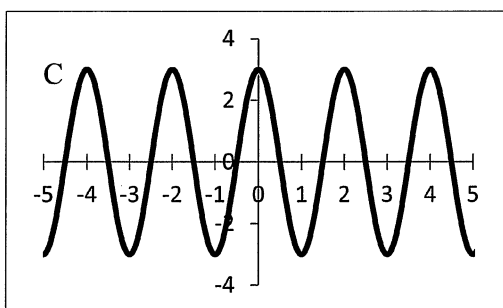
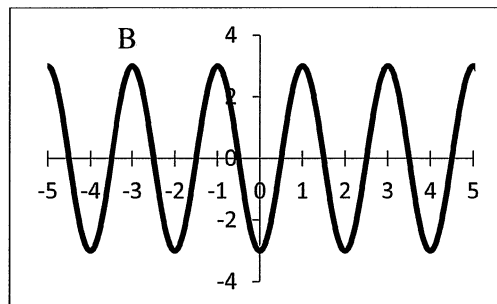
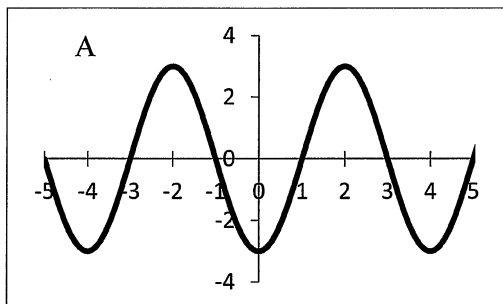
Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

1. Find the Period and Phase Shift of the equation  $y = \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$ .

not on  
Ex. 2  
sp'14

- A. Period =  $4\pi$ , Phase Shift =  $\frac{\pi}{6}$   
B. Period =  $4\pi$ , Phase Shift =  $\frac{2\pi}{3}$   
C. Period =  $\pi$ , Phase Shift =  $\frac{\pi}{6}$   
D. Period =  $\pi$ , Phase Shift =  $\frac{2\pi}{3}$   
E. None of the above

2. Which of the following graphs best represents the equation  $y = 3\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$ ?



not on  
Ex. 2  
sp'14

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

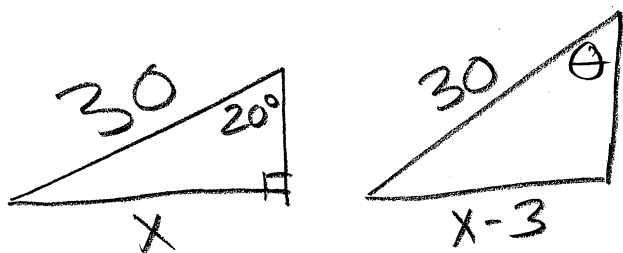
3. Scientists sometimes use the formula  $f(t) = a \sin(bt + c) + d$  to simulate temperature variations during the days, with time  $t$  in hours, temperature  $f(t)$  in  $^{\circ}\text{C}$ , and  $t = 0$  corresponding to midnight. Assume that  $f(t)$  is decreasing at midnight.

On a winter day last year, the high temperature was  $10^{\circ}\text{C}$ , and the low of temperature of  $4^{\circ}\text{C}$  occurred at 3 am. Which of the following statements best describes the temperature at noon ( $t=12$ )?

- A. Between  $7^{\circ}\text{C}$  and  $10^{\circ}\text{C}$  and rising  
 B. Between  $3^{\circ}\text{C}$  and  $7^{\circ}\text{C}$  and falling  
 C. Between  $3^{\circ}\text{C}$  and  $7^{\circ}\text{C}$  and rising  
 D. Between  $7^{\circ}\text{C}$  and  $10^{\circ}\text{C}$  and falling  
 E. None of the above.

Not on  
 Ex. 2  
 Sp '14

4. A 30 foot ladder is leaning against the side of a building such that there is a  $20^{\circ}$  between the ladder and the building. If the bottom of the ladder is moved three feet **closer** to the base of the building, what angle will the ladder now make with the building? Please round to the nearest tenth of a degree.



- A.  $15.6^{\circ}$   
 B.  $14.0^{\circ}$   
 C.  $16.2^{\circ}$   
 D.  $14.8^{\circ}$   
 E. None of the above

$$\sin 20^{\circ} = \frac{X}{30}$$

$$X = 30 \sin 20^{\circ}$$

$$X = 10.26$$

$$X - 3 = 7.26$$

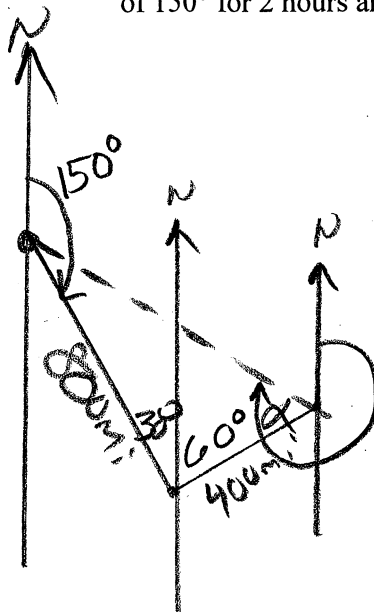
$$\sin \theta = \frac{7.26}{30}$$

$$\theta = \sin^{-1}\left(\frac{7.26}{30}\right)$$

$$\theta = 14.0^{\circ}$$

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

For questions 5 and 6: An airplane, flying at a speed of 400 miles per hour, flies in the direction of  $150^\circ$  for 2 hours and then flies in the direction  $60^\circ$  for one hour.



$$D = rt$$

$$D_1 = 400(2) = 800$$

$$D_2 = 400(1) = 400$$

$$180^\circ - 150^\circ = 30^\circ$$

$$30^\circ + 60^\circ = 90^\circ$$

Rt.  $\Delta$

5. Approximate, to the nearest mile, how far the plane is from point A.

$$D^2 = 800^2 + 400^2$$

$$D = 894 \text{ mi}$$

If it A3K for TIME BACK

$$t = \frac{D}{r} = \frac{894.4272}{400} = 2.2 \text{ hrs}$$

A. 953 miles

B. 862 miles

C. 916 miles

D. 894 miles

E. None of the above

6. Approximate, to the nearest degree, what direction the plane needs to fly in order to return to point A.

$$N \text{ to } S = 180^\circ$$

$$Alt \text{ int Angle} = 60^\circ$$

$$\tan \alpha = \frac{800}{400}$$

$$\alpha = \tan^{-1}(2)$$

$$\alpha = 63^\circ$$

$$\begin{array}{r} 180^\circ \\ + 60^\circ \\ + 63^\circ \\ \hline \end{array}$$

$$303^\circ$$

A.  $303^\circ$

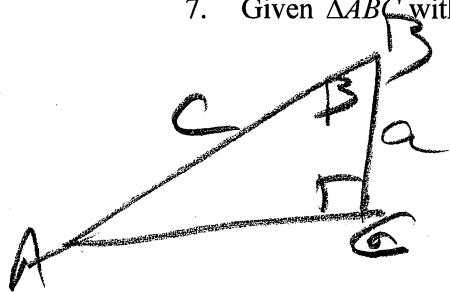
B.  $311^\circ$

C.  $287^\circ$

D.  $295^\circ$

E. None of the above

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

7. Given  $\triangle ABC$  with  $\gamma = 90^\circ$ , express side  $c$  in terms of angle  $\beta$  and side  $a$ .

$$\cos \beta = \frac{a}{c}$$

$$c = \frac{a}{\cos \beta}$$

$$c = a \sec \beta$$

A.  $c = a \tan \beta$

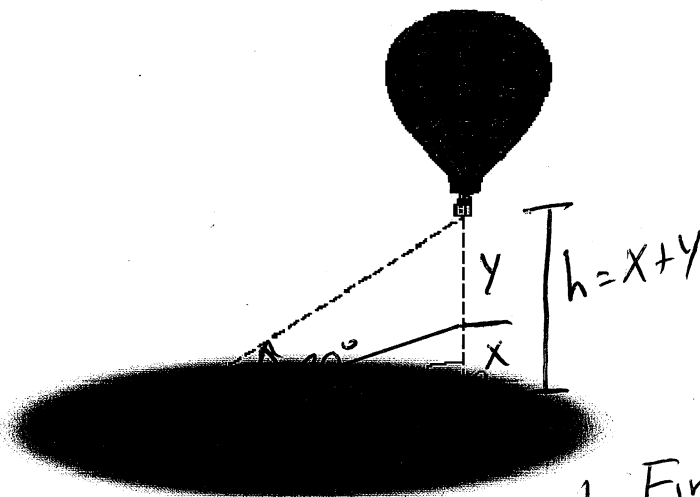
B.  $c = a \csc \beta$

C.  $c = a \sin \beta$

D.  $c = a \sec \beta$

E.  $c = a \cos \beta$

8. As a hot-air balloon rises vertically, its angle of elevation from a point  $P$  on level ground  $d = 200$  kilometers from the point  $Q$  directly underneath the balloon changes from  $19^\circ$  to  $30^\circ$  (see the figure). Approximately how far does the balloon rise during this period? (Give the answer to one decimal place.)



A. 34.9 km

B. 38.2 km

C. 46.6 km

D. 41.7 km

E. None of the above

$$1. \tan 19^\circ = \frac{x}{200}$$

$$x = 200 \tan 19^\circ$$

$$x = 68.8655$$

$$2. \tan 30^\circ = \frac{h}{200}$$

$$h = 200 \tan 30^\circ$$

$$h = 115.4701$$

1. Find  $x$ 2. Find  $h$ 3. Subtract to find  $y$ 

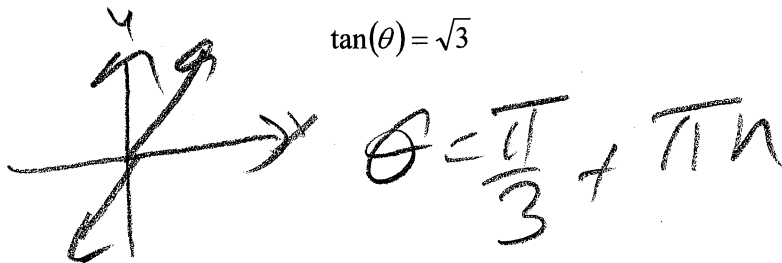
$$y = 115.4701 - 68.8655$$

$$y = \underline{\underline{46.6 \text{ km}}}$$

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25.

9. Find all solutions of the equation using  $n$  as an arbitrary integer.

$$\tan(\theta) = \sqrt{3}$$



A.  $\theta = \frac{\pi}{4} + \pi n$

B.  $\theta = \frac{\pi}{6} + \pi n$

C.  $\theta = \frac{2\pi}{3} + \pi n$

D.  $\theta = \frac{5\pi}{6} + \pi n$

E. None of the above

10. Find all solutions of the equation using  $n$  as an arbitrary integer.

$$\cos\left(2\phi - \frac{\pi}{4}\right) = 0$$



$$2\phi - \frac{\pi}{4} = \frac{\pi}{2} + \pi n$$

$$2\phi = \frac{\pi}{4} + \frac{\pi}{2} + \pi n$$

$$2\phi = \frac{3\pi}{4} + \pi n$$

$$\phi = \frac{3\pi}{8} + \frac{\pi}{2}n$$

A.  $\phi = \frac{\pi}{8} + \pi n$

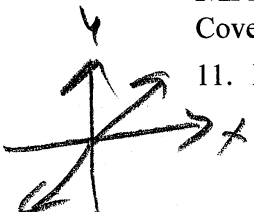
B.  $\phi = \frac{3\pi}{8} + \frac{\pi}{2}n$

C.  $\phi = \frac{\pi}{8} + \frac{\pi}{2}n$

D.  $\phi = \frac{3\pi}{8} + \pi n$

E. None of the above

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

11. Find all solutions of the equation in the interval  $[0, 2\pi)$ .

$$\tan\left(2\mu - \frac{\pi}{2}\right) = 1$$

$$2\mu - \frac{\pi}{2} = \frac{\pi}{4} + \pi n$$

$$2\mu = \frac{3\pi}{4} + \pi n$$

$$\mu = \frac{3\pi}{8} + \frac{\pi}{2}n$$

rewrite with common Den.

$$\mu = \frac{3\pi}{8} + \frac{4\pi}{8}n$$

| n  | μ         |
|----|-----------|
| 0  | 3π/8      |
| 1  | 7π/8      |
| 2  | 11π/8     |
| 3  | 15π/8     |
| 4  | TOO BIG   |
| -1 | TOO SMALL |

A.  $\mu = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$

B.  $\mu = \frac{\pi}{3}, \frac{4\pi}{3}$

C.  $\mu = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

D.  $\mu = \frac{3\pi}{8}, \frac{11\pi}{8}$

E. None of the above

12. Express as a trigonometric function of one angle.

$$\sin(54^\circ)\cos(14^\circ) - \cos(54^\circ)\sin(14^\circ)$$

$$\sin(54^\circ - 14^\circ)$$

$$\sin(40^\circ)$$

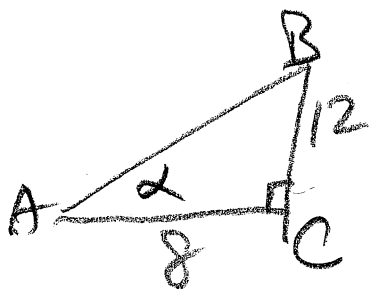
A.  $\sin(40^\circ)$

B.  $\cos(40^\circ)$

C.  $\sin(68^\circ)$

D.  $\cos(68^\circ)$

E. None of the above

13. Given  $\triangle ABC$  with  $\gamma = 90^\circ$ ,  $a = 12$ , and  $b = 8$ , approximate angle  $\alpha$  to the nearest tenth of a degree.

$$\tan \alpha = \frac{12}{8}$$

$$\alpha = \tan^{-1}\left(\frac{12}{8}\right)$$

$$\alpha = 56.3^\circ$$

A.  $\alpha = 33.7^\circ$

B.  $\alpha = 41.8^\circ$

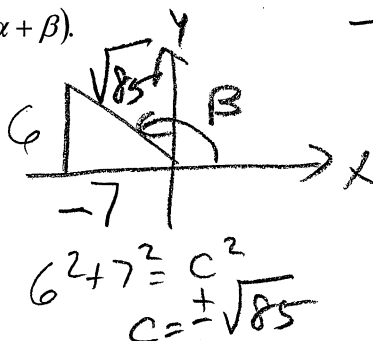
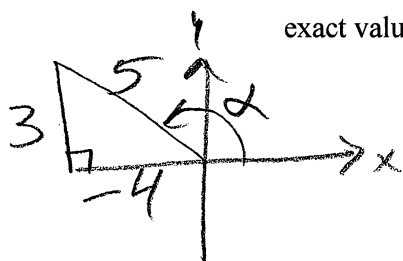
C.  $\alpha = 56.3^\circ$

D.  $\alpha = 48.2^\circ$

E. None of the above

Covers Lessons 10-19, Sections 6.5, 6.7, 7.2, 7.3 and 7.4 up to question #25

14. If  $\sin \alpha = \frac{3}{5}$  and  $\tan \beta = \frac{-6}{7}$  for a second-quadrant angles  $\alpha$  and  $\beta$ , then find the exact value of  $\cos(\alpha + \beta)$ .



$$\tan \beta = \frac{y}{x} = \frac{6}{-7}$$

A.  $\frac{15}{5\sqrt{85}}$

B.  $\frac{2}{\sqrt{85}}$

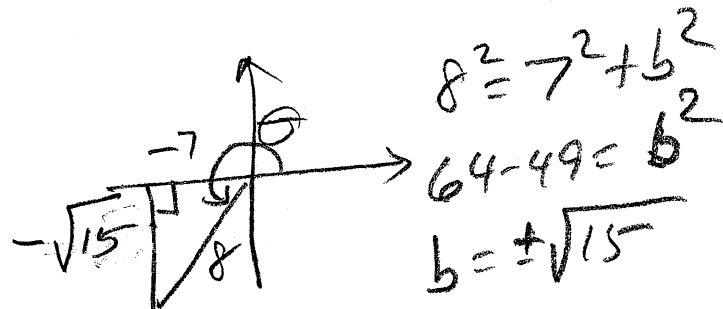
C.  $\frac{36}{5\sqrt{85}}$

D.  $\frac{-15}{5\sqrt{85}}$

E.  $\frac{-2}{\sqrt{85}}$

$$\begin{aligned}\cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(\frac{-4}{5}\right)\left(\frac{-7}{\sqrt{85}}\right) - \left(\frac{3}{5}\right)\left(\frac{6}{\sqrt{85}}\right) \\ &= \frac{28}{5\sqrt{85}} - \frac{18}{5\sqrt{85}} = \frac{10}{5\sqrt{85}} = \frac{2}{\sqrt{85}}\end{aligned}$$

15. Find the exact value of  $\sin(2\theta)$  if  $\cos \theta = \frac{-7}{8}$  and  $180^\circ < \theta < 270^\circ$ .



A.  $\frac{-17}{32}$

B.  $\frac{17}{32}$

C.  $\frac{-7\sqrt{15}}{32}$

D.  $\frac{7\sqrt{15}}{32}$

E. None of the above

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\begin{aligned}&= 2\left(\frac{-\sqrt{15}}{8}\right)\left(\frac{-7}{8}\right) \\ &= \frac{14\sqrt{15}}{64} = \frac{7\sqrt{15}}{32}\end{aligned}$$



| Question | Letter | Answer   |
|----------|--------|--|
| 1.       | B      | Period = $4\pi$ , Phase Shift = $\frac{2\pi}{3}$                         |
| 2.       | D      | See Exam   |
| 3.       | A      | Between $7^{\circ}\text{C}$ and $10^{\circ}\text{C}$ and rising          |
| 4.       | B      | $14.0^{\circ}$   |
| 5.       | D      | 894 miles  |
| 6.       | A      | $303^{\circ}$  |
| 7.       | D      | $c = a \sec \beta$   |
| 8.       | C      | 46.6 km  |
| 9.       | E      | $\theta = \frac{\pi}{3} + \pi n$ (none of the above)                     |
| 10.      | B      | $\phi = \frac{3\pi}{8} + \frac{\pi}{2}n$                                 |
| 11.      | C      | $\mu = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ |
| 12.      | A      | $\sin(40^{\circ})$   |
| 13.      | C      | $\alpha = 56.3^{\circ}$  |
| 14.      | B      | $\frac{2}{\sqrt{85}}$  |
| 15.      | D      | $\frac{7\sqrt{15}}{32}$  |