

MA 15400

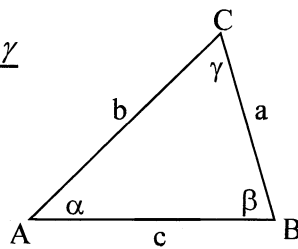
Fall 2012

Exam 3

*With 2 questions
from Ex. 2
Fall '12*

LAW OF SINES:

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



LAW OF COSINES:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

~~Covers Lessons 12 to 22, Section 6.7, 7.2, 7.3, 7.4, and 7.6 (questions 1 to 34)~~

14. Find the exact value of the expression whenever it is defined.

$$\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

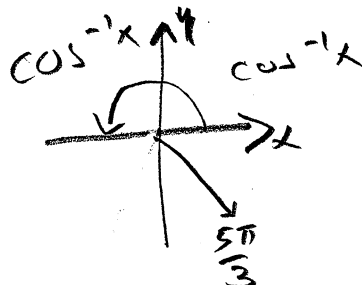
A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $-\frac{\pi}{3}$

D. $\frac{5\pi}{3}$

E. None of the above



15. Write the expression as an algebraic expression in x for $x > 0$.

A. $\frac{x}{2}$

B. $\frac{x-x^2}{2}$

C. $\frac{4x-x^2}{8}$

D. $\frac{x\sqrt{16-x^2}}{8}$

$$\sin\left(2\sin^{-1}\left(\frac{x}{4}\right)\right)$$

α

$$\sin(2\alpha)$$

$$= 2\sin\alpha \cos\alpha$$

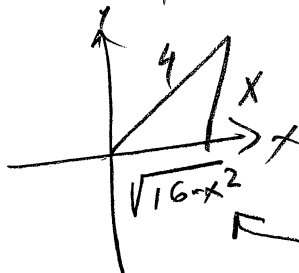
$$= 2\left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{4}\right)$$

$$= \frac{2x\sqrt{16-x^2}}{16}$$

$$= \frac{x\sqrt{16-x^2}}{8}$$

$$\alpha = \sin^{-1}\left(\frac{x}{4}\right)$$

$$\sin\alpha = \frac{x}{4}$$



$$4^2 = x^2 + a^2$$

$$16 - x^2 = a^2$$

$$\sqrt{16-x^2} = a$$

Covers Lessons 23 to 32, Sections 7.6 (questions 53-67), 8.1, 8.2, and 8.3

1. Find $2a + 3b$ for $a = \langle 3, -5 \rangle$ and $b = \langle -6, 8 \rangle$

A. $\langle -6, 9 \rangle$

B. $\langle -3, 1 \rangle$

C. $\langle -12, 14 \rangle$

D. $\langle -9, 6 \rangle$

E. None of the above

$$\begin{array}{r} 2a = \langle 6, -10 \rangle \\ + 3b = \langle -18, 24 \rangle \\ \hline \langle -12, 14 \rangle \end{array}$$

2. Approximate the solutions of the equation, to four decimal places, that are in the interval $[0, 2\pi)$.

$6\sin x \cos x = 7\cos x$

A. $x = 1.3808, 4.6035$

B. $x = 1.2489, 3.9521$

C. $x = 1.2489, 4.6035$

D. $x = 1.3808, 3.9521$

E. None of the above

$6\sin x \cos x - 7\cos x = 0$

$\cos x (6\sin x - 7) = 0$

$\cos x = 0 \quad 6\sin x - 7 = 0$

$\sin x = 7/6$

no solution

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 1.5708, 4.7124$

3. Approximate the solutions of the equation, to four decimal places, that are in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

$2\tan^2 t + 9\tan t - 5 = 0$

A. $t = 1.2032, -0.5758$

B. $t = 0.4636, -1.3734$

C. $t = 2.5969, -0.6492$

D. $t = 0.5989, -1.4731$

E. None of the above

$a = 2 \quad b = 9 \quad c = -5$

$$\tan t = \frac{-9 \pm \sqrt{81 - 4(2)(-5)}}{2(2)} = \frac{-9 \pm \sqrt{121}}{4}$$

$\tan t = \frac{-9 + 11}{4}$

$\tan t = 1/2$

$t = \tan^{-1}(1/2)$

$t = 0.4636$

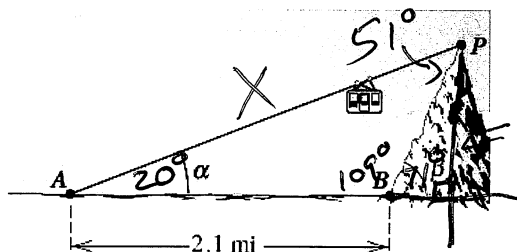
$\tan t = \frac{-9 - 11}{4}$

$\tan t = -5$

$t = \tan^{-1}(-5)$

$t = -1.3734$

- 4 and 5: As shown in the figure below, a cable car carries passengers from a point A , which is 2.1 miles from a point B at the base of a mountain, to a point P at the top of the mountain. The angles of elevation of P from A and B are $\alpha = 20^\circ$ and $\beta = 71^\circ$, respectively.



Check the mode on your calculator!
Check it again!
One more time!

$$\begin{array}{r} 180^\circ \\ - 71^\circ \\ \hline 109^\circ \end{array} \quad \begin{array}{r} 109^\circ \\ + 20^\circ \\ \hline 129^\circ \end{array} \quad \begin{array}{r} 180^\circ \\ - 129^\circ \\ \hline 51^\circ \end{array}$$

4. Approximate, to two decimal places, the distance between A and P .

A. 2.55 miles

B. 2.64 miles

C. 2.86 miles

D. 2.71 miles

E. None of the above

$$\frac{\sin 109^\circ}{X} = \frac{\sin 51^\circ}{2.1}$$

$$X = \frac{2.1 \sin 109^\circ}{\sin 51^\circ}$$

$$X = 2.554976$$

5. Approximate, to two decimal places, the height of the mountain.

A. 0.93 miles

B. 0.87 miles

C. 0.98 miles

D. 0.90 miles

E. None of the above

$$\sin 20^\circ = \frac{h}{2.554976}$$

$$2.554976 \sin 20^\circ = h$$

$$h = 0.87$$

Covers Lessons 23 to 32, Sections 7.6 (questions 53-67), 8.1, 8.2, and 8.3

6. A surveyor notes that the direction from point A to point B is $N57^\circ W$ and the direction from A to point C is $S17^\circ W$. The distance from A to B is 209 yards, and the distance from B to C is 354 yards. Approximate the distance from A to C to the nearest whole yard.

A. 292 yds.

B. 349 yds.

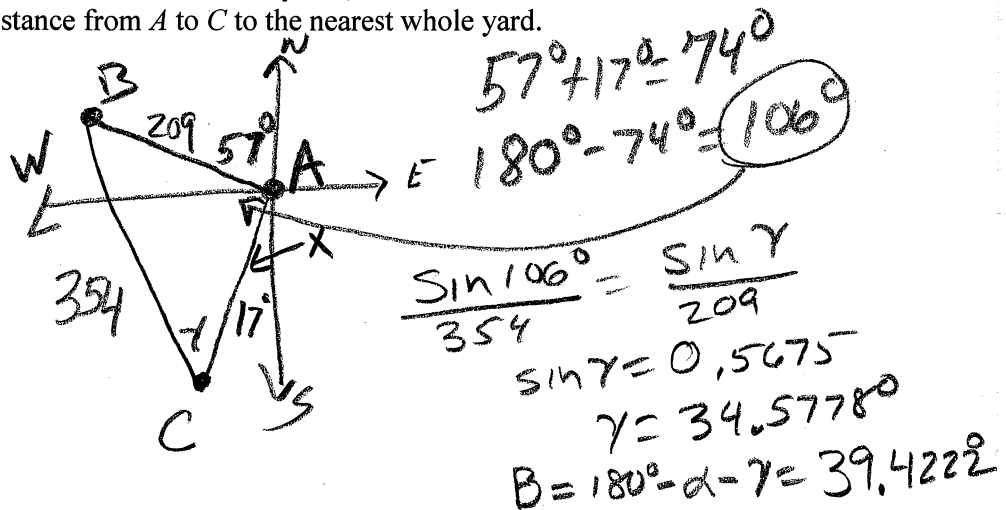
C. 234 yds.

D. 286 yds.

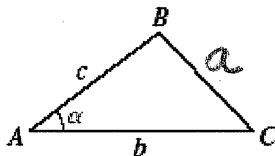
E. None of the above

$$\frac{\sin 106^\circ}{354} = \frac{\sin 39.42^\circ}{x}$$

$$x = 233.86$$



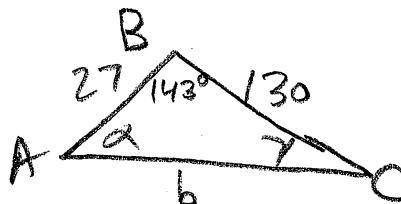
7. Given the indicated parts of $\triangle ABC$, what **angle or side** would you find next, and what would you use to find it?

A. Angle γ , Law of SinesB. Side a , Law of CosinesC. Angle β , Law of SinesD. Side a , Law of SinesE. Angle γ , Law of Cosines

8. Solve $\triangle ABC$ for $\beta = 143^\circ$, $a = 130$, and $c = 27$. Approximate the values to one decimal place.

A. $\gamma = 6.1^\circ$, $\alpha = 30.9^\circ$, $b = 152.4$ B. $\gamma = 8.5^\circ$, $\alpha = 28.5^\circ$, $b = 109.6$ C. $\gamma = 7.1^\circ$, $\alpha = 29.9^\circ$, $b = 131.0$ D. $\gamma = 6.6^\circ$, $\alpha = 30.4^\circ$, $b = 141.7$

E. None of the above



$$b^2 = 130^2 + 27^2 - 2(27)(130)\cos 143^\circ$$

$$b = 152.4$$

$$\frac{\sin 143^\circ}{152.4} = \frac{\sin \alpha}{130}$$

$$\alpha = 30.8807^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

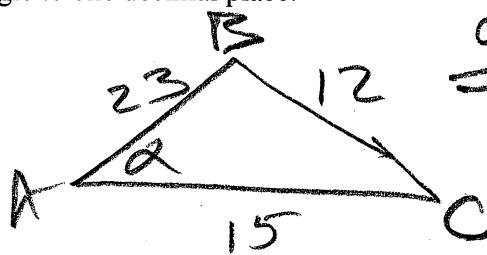
$$\gamma = 6.1^\circ$$

Covers Lessons 23 to 32, Sections 7.6 (questions 53-67), 8.1, 8.2, and 8.3

9. Given $\triangle ABC$, with $a=12$, $b=15$, and $c=23$. Approximate the value of the smallest angle in the triangle to one decimal place.

A. 24.0° B. 35.7° C. 27.9° D. 31.8°

E. None of the above

 α smallest

$$12^2 = 23^2 + 15^2 - 2(23)(15)\cos\alpha$$

$$\frac{144 - 529 - 225}{-2(23)(15)} = \cos\alpha$$

$$0.8841 = \cos\alpha$$

$$\alpha = 27.86^\circ$$

10. A ship leaves port at 1:00 P.M. and travels $S35^\circ E$ at the rate of 25 mph. Another ship leaves the same port at the same time, and travels $S20^\circ W$ at 20 mph. To the nearest tenth of a mile, approximately how far apart are the ships at 4:00 P.M.?

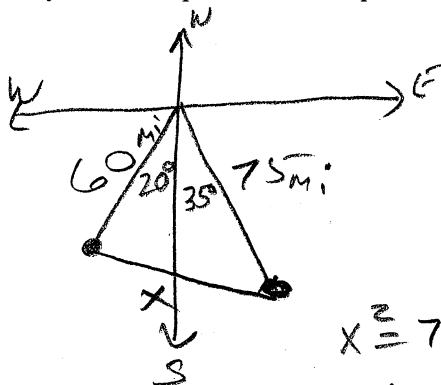
A. 63.7 miles

B. 96.0 miles

C. 119.9 miles

D. 45.0 miles

E. None of the above



$$3 \text{ hr } D = rt$$

$$D_1 = 25(3) = 75$$

$$D_2 = 20(3) = 60$$

$$20^\circ + 35^\circ = 55^\circ$$

$$x^2 = 75^2 + 60^2 - 2(75)(60)\cos 55^\circ$$

$$x = 63.7402$$

11. Find the magnitude of the vector a and the smallest positive angle θ from the positive x -axis to the vector that corresponds to a . Round your answers to one decimal.

$$a = -8i + 2j$$

$$a = \langle -8, 2 \rangle$$

A. $\|a\| \approx 4.5$, $\theta \approx 166.0^\circ$ B. $\|a\| \approx 8.2$, $\theta \approx 104.0^\circ$ C. $\|a\| \approx 4.5$, $\theta \approx 104.0^\circ$ D. $\|a\| \approx 8.2$, $\theta \approx 166.0^\circ$

E. None of the above

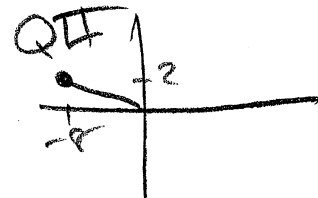
$$\|a\| = \sqrt{8^2 + 2^2} = \sqrt{68}$$

$$\|a\| = 8.2$$

$$\tan \theta = \frac{y}{x} = \frac{2}{-8}$$

$$\theta = \tan^{-1}(-1/4) + 180^\circ \text{ [QII Vector]}$$

$$\theta = 165.9638^\circ$$



Covers Lessons 23 to 32, Sections 7.6 (questions 53-67), 8.1, 8.2, and 8.3

12. The vectors \mathbf{a} and \mathbf{b} represent two forces acting at the same point, and θ is the smallest positive angle between \mathbf{a} and \mathbf{b} . Approximate the magnitude of the resultant force to one decimal place.

$$\|\mathbf{a}\| = 5.9 \text{ lb.}, \|\mathbf{b}\| = 6.8 \text{ lb.}, \theta = 80^\circ$$

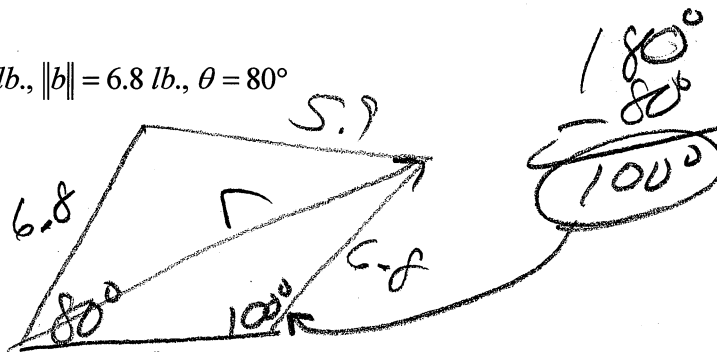
A. $\|\mathbf{r}\| \approx 9.4 \text{ lb.}$

B. $\|\mathbf{r}\| \approx 10.4 \text{ lb.}$

C. $\|\mathbf{r}\| \approx 9.7 \text{ lb.}$

D. $\|\mathbf{r}\| \approx 8.2 \text{ lb.}$

E. None of the above



$$r^2 = 5.9^2 + 6.8^2 - 2(5.9)(6.8)\cos 100^\circ$$

$$\|\mathbf{r}\| = 9.7459$$

13. The given information about $\triangle ABC$, $\gamma = 64^\circ$, $c = 23$ and $b = 25$, creates two triangles. Find the perimeter of both triangles, rounded to one decimal place.

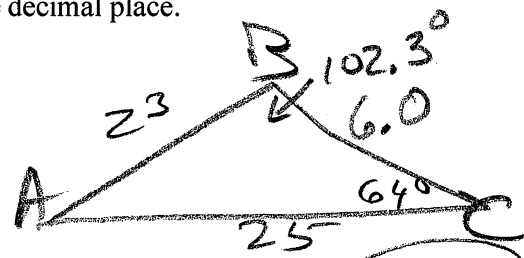
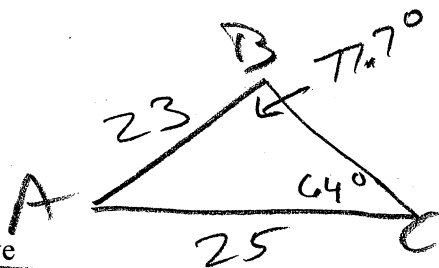
A. 65.5, 55.9

B. 64.9, 52.4

C. 62.7, 53.6

D. 63.9, 54.0

E. None of the above



$$P_2 = 54.0$$

$$P_1 = 63.9$$

$$\frac{\sin 64^\circ}{23} = \frac{\sin B}{25}$$

$$\gamma = 64^\circ \quad \sin B = 0.9770$$

$$B_1 = 77.6743^\circ$$

$$\alpha_1 = 38.3257^\circ$$

$$\frac{\sin 64^\circ}{23} = \frac{\sin 38.33^\circ}{a_1}$$

$$a_1 = 15.8690$$

$$\gamma = 64^\circ \quad B_2 = 102.3257^\circ$$

$$\alpha_2 = 13.6743^\circ$$

$$\frac{\sin 64^\circ}{23} = \frac{\sin 13.67^\circ}{a_2}$$

$$a_2 = 6.0495$$

Covers Lessons 23 to 32, Sections 7.6 (questions 53-67), 8.1, 8.2, and 8.3

14. Find a vector of **magnitude 6** that has the **opposite** direction of $a = -5i + 12j$

A. $30i - 72j$

B. $\frac{30}{13}i - \frac{72}{13}j$

C. $-30i + 72j$

D. $-\frac{30}{13}i + \frac{72}{13}j$

E. None of the above

Find unit vector

$$-6u$$

$$||a|| = \sqrt{5^2 + 12^2} = 13$$

$$u = \frac{1}{13}(-5i + 12j)$$

$$u = -\frac{5}{13}i + \frac{12}{13}j$$

$$-6u = -6\left(-\frac{5}{13}i + \frac{12}{13}j\right)$$

$$-6u = \frac{30}{13}i - \frac{72}{13}j$$

15. An airplane is flying in the direction 100° with airspeed of 400 mph, and a 30 mph wind is blowing in the direction 25° . Approximate the true course and ground speed of the airplane to the nearest whole numbers.

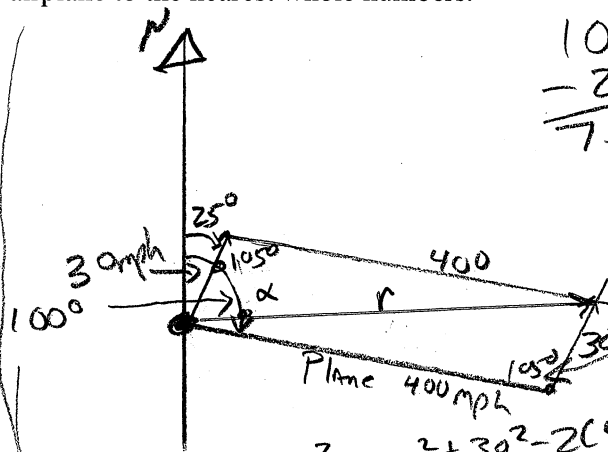
A. 393 mph, 72°

B. 409 mph, 72°

C. 393 mph, 96°

D. 409 mph, 96°

E. None of the above

Vector Method:
 $P + W = R$ 

$$\begin{array}{r} 100^\circ \\ - 25^\circ \\ \hline 75^\circ \end{array} \quad \begin{array}{r} 1800 \\ - 756 \\ \hline 1044 \end{array}$$

$$R^2 = 400^2 + 30^2 - 2(400)(30)\cos 105^\circ$$

$$||R|| = 408.7929 \text{ mph}$$

$$\frac{\sin \alpha}{400} = \frac{\sin 105^\circ}{408.8}$$

$$\alpha = 70.9351^\circ$$

$$25^\circ + 71^\circ = 96^\circ$$

$$P: \langle 400 \cos 100^\circ, 400 \sin 100^\circ \rangle$$

$$W: \langle 30 \cos 25^\circ, 30 \sin 25^\circ \rangle$$

$$R: \langle -42.27, 406.60 \rangle$$

$$||R|| = \sqrt{x^2 + y^2} = 408.7929 \text{ mph}$$

$$\tan \theta = \frac{y}{x} = \frac{406.60}{-42.27}$$

$$\theta = \tan^{-1}(-9.6191) + 180^\circ$$

$$\theta = -84.10 + 180^\circ$$

$$\theta = 95.9351^\circ$$

+180°
Since QII
vector

Exam 3 Answers

	Answer	Letter
1.	$\langle -12, 14 \rangle$	C
2.	$x = 1.5708, 4.7124$	E
3.	$t = 0.4636, -1.3734$	B
4.	2.55 miles (Too close to 2.56, sorry)	A or E
5.	0.87 miles	B
6.	234 yds.	C
7.	Side a , Law of Cosines	B
8.	$\gamma = 6.1^\circ, \alpha = 30.9^\circ, b = 152.4$	A
9.	27.9°	C
10.	63.7 miles	A
11.	$\ a\ \approx 8.2, \theta \approx 166.0^\circ$	D
12.	$\ r\ \approx 9.7 \text{ lb.}$	C
13.	$63.9, 54.0$	D
14.	$\frac{30}{13}i - \frac{72}{13}j$	B
15.	$409 \text{ mph}, 96^\circ$	D