

MA 15400

Spring 2012

Exam 3

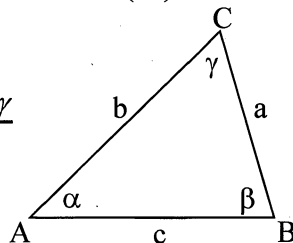
$$\sin(2u) = 2 \sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

LAW OF SINES

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

ANGLE BETWEEN TWO VECTORS:

$$\cos \theta = \frac{(\vec{a}) \cdot (\vec{b})}{\|\vec{a}\| \|\vec{b}\|}$$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

1. Given $a = \langle -2, 6 \rangle$ and $b = \langle 4, 5 \rangle$, find $2a - 3b$.

A. $\langle -8, 27 \rangle$

B. $\langle -16, -3 \rangle$

C. $\langle 8, 3 \rangle$

D. $\langle 16, -27 \rangle$

E. None of the above

$$2a = \langle -4, 12 \rangle$$

$$3b = \langle 12, 15 \rangle$$

$$\underline{\hspace{1cm}}$$
$$\langle -16, -3 \rangle$$

2. Find the solutions of the equation that are in the interval $[0, 2\pi)$.

$$\sqrt{2} \sin t + \sin(2t) = 0$$

A. $0, \pi, \frac{\pi}{4}, \frac{7\pi}{4}$

B. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}$

C. $0, \pi, \frac{3\pi}{4}, \frac{5\pi}{4}$

D. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{7\pi}{4}$

E. None of the above

Not on
Ex. 3
This semester

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

3. Vectors a and b represent two forces acting at the same point, and θ is the smallest positive angle between a and b . Approximate the magnitude of the resultant force to the nearest tenth of a pound.

$$a = 4.6 \text{ lb}, b = 7.8 \text{ lb}, \theta = 71^\circ$$

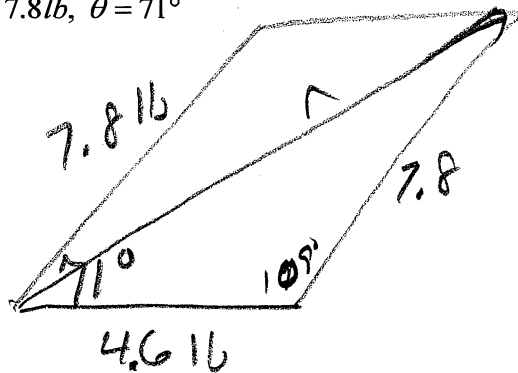
A. 9.7 lb

B. 7.7 lb

C. 9.0 lb

D. 10.3 lb

E. None of the above.



$$\begin{array}{r} 180 \\ - 71 \\ \hline 109^\circ \end{array}$$

$$r^2 = 4.6^2 + 7.8^2 - 2(4.6)(7.8)\cos 109^\circ$$

$$\|r\| = 10.2646$$

4. Find the exact value of the expression.

$$\sin^{-1}\left(\sin \frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

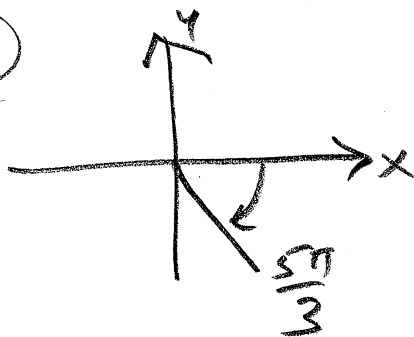
A. $-\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{4\pi}{3}$

E. None of the above



not $\frac{5\pi}{3}$ since

$\frac{5\pi}{3}$ is out of

Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

5. Find the exact value of the expression.

$$\sin \left[2 \arccos \left(\frac{-4}{7} \right) \right] \longrightarrow \sin 2\alpha$$

A. $\frac{-25}{49}$

B. $\frac{8\sqrt{33}}{49}$

C. $\frac{25}{49}$

D. $\frac{-8\sqrt{33}}{49}$

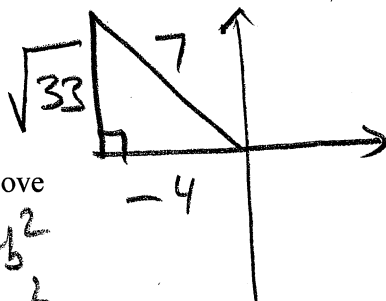
E. None of the above

$$\alpha = \cos^{-1} \left(\frac{-4}{7} \right)$$

$$= 2 \sin \alpha \cos \alpha$$

$$= 2 \left(\frac{\sqrt{33}}{7} \right) \left(\frac{-4}{7} \right)$$

$$= \frac{-8\sqrt{33}}{49}$$



$$\begin{aligned} 7^2 &= 4^2 + b^2 \\ 49 - 16 &= b^2 \\ b &= \pm \sqrt{33} \end{aligned}$$

6. Determine m such that the two vectors are orthogonal.

$$a = \langle 3m, 6 \rangle, b = \langle 5, -7 \rangle$$

A. $m = \frac{35}{18}$

B. $m = \frac{14}{5}$

C. $m = \frac{-35}{18}$

D. $m = \frac{-14}{5}$

E. None of the above

$$a \cdot b = (3m)(5) + (6)(-7)$$

$$a \cdot b = 15m - 42$$

$$15m - 42 = 0$$

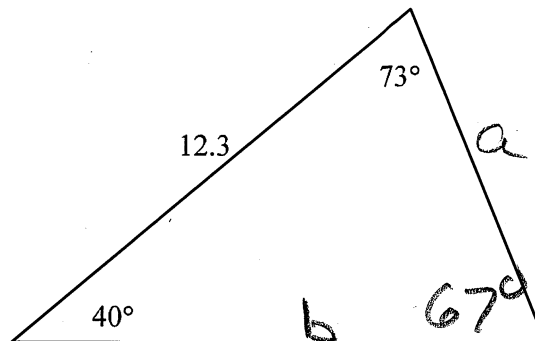
$$15m = 42$$

$$m = \frac{42}{15} = \frac{14}{5}$$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

7. Approximate the perimeter of the given triangle to one decimal place.

- A. 35.4
B. 28.7
C. 33.7
D. 30.4
E. None of the above



$$180^\circ - (73^\circ + 40^\circ)$$

$$= 67^\circ$$

$$\frac{\sin 67^\circ}{12.3} = \frac{\sin 40^\circ}{a}$$

$$a = 8.5891$$

$$\frac{\sin 67^\circ}{12.3} = \frac{\sin 73^\circ}{b}$$

$$b = 12.7784$$

$$P = 12.3 + 8.5891 + 12.7784 = 33.6674$$

8. An airplane, with airspeed of 250 miles per hour, is flying in the direction 65° and a 52 mile per hour wind is blowing directly from the west. What is the ground speed of the airplane rounded to the nearest mile per hour?

- A. 285 mph
B. 294 mph
C. 276 mph
D. 298 mph

E. None of the above

Vector Method:
 $P + W = R$

$$P: \langle 250 \cos 65^\circ, 250 \sin 65^\circ \rangle$$

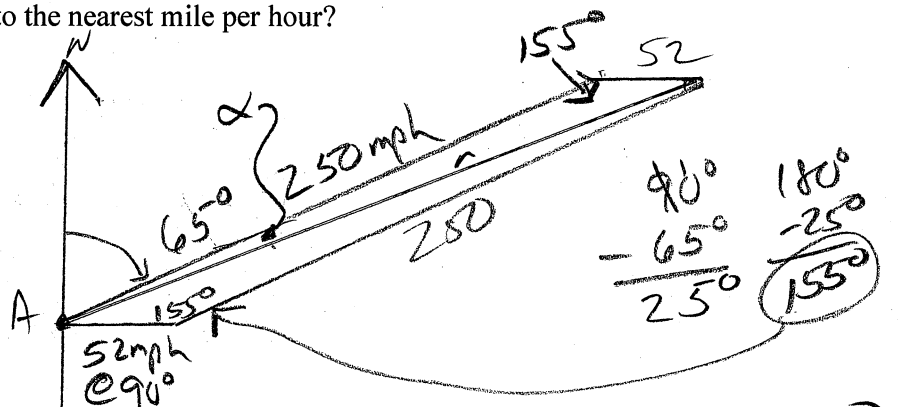
$$+ W: \langle 52 \cos 90^\circ, 52 \sin 90^\circ \rangle$$

$$R: \langle 105.65, 278.58 \rangle$$

$$\|R\| = \sqrt{x^2 + y^2} = 297.9396$$

$$\tan \theta = \frac{y}{x} = \frac{278.58}{105.65} = 2.64$$

$$\theta = 69.23^\circ \text{ True Course}$$



$$r^2 = 250^2 + 52^2 - 2(250)(52)\cos 155^\circ$$

$$\|r\| = 297.9396 \text{ Ground Speed}$$

If it also Ask for True Course:

$$\frac{\sin \alpha}{52} = \frac{\sin 155^\circ}{297.9396}$$

$$\sin \alpha = 0.0738$$

$$\alpha = 4.23^\circ$$

$$\text{True Course: } 65^\circ + 4^\circ = 69^\circ$$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

9. The bearing from Point A to Point B is $N37^\circ W$ and the bearing from Point A to Point C is $N21^\circ E$. The distance from Point A to Point B is 175 miles and the distance from Point A to Point C is 222 miles. To the nearest mile, approximately how far is it from Point B to Point C?

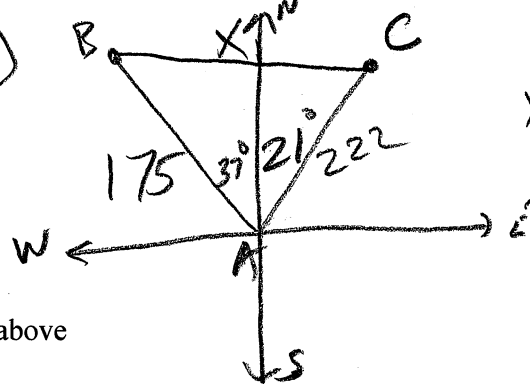
A. 197 miles

B. 348 miles

C. 217 miles

D. 309 miles

E. None of the above



$$37^\circ + 21^\circ = 58^\circ$$

$$X^2 = 175^2 + 222^2 - 2(175)(222)\cos 58^\circ$$

$$X = 196.8102$$

10. From Point P, the angle of elevation of the top of a nearby building is 29° . From a point 350 feet closer to the building, and on the line connecting Point P and the base of the building, the angle of elevation to the top of the same building is 43° . Rounded to the nearest foot, what is the height of the building?

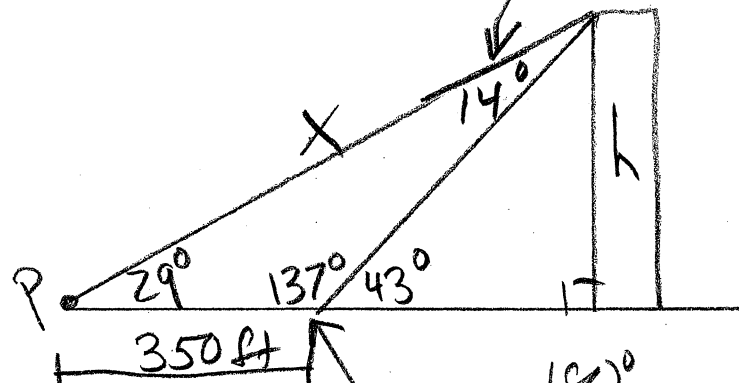
A. 987 feet

B. 701 feet

C. 478 feet

D. 566 feet

E. None of the above



$$\frac{\sin 137^\circ}{X} = \frac{\sin 14^\circ}{350}$$

$$X = 986.6797$$

$$\sin 29^\circ = \frac{h}{X}$$

$$986.7 \sin 29^\circ = h = 478.35$$

$$\begin{array}{r} 180^\circ \\ - 43^\circ \\ \hline 137^\circ \end{array}$$

$$\begin{array}{r} 180^\circ \\ - 29^\circ \\ - 137^\circ \\ \hline 14^\circ \end{array}$$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

11. Find the angle between vectors $a = \langle 4, 2 \rangle$ and $b = \langle -5, 3 \rangle$ to the nearest tenth of a degree.

A. 105.6° B. 122.5° C. 100.1° D. 109.4°

E. None of the above

$$a \cdot b = (4)(-5) + (2)(3) = -20 + 6 = -14$$

$$\|a\| = \sqrt{16+4} = \sqrt{20}$$

$$\|b\| = \sqrt{25+9} = \sqrt{34}$$

$$\cos \theta = \frac{-14}{\sqrt{20} \sqrt{34}} = \frac{-14}{\sqrt{680}}$$

$$\cos \theta = -0.5389$$

$$\theta = 122.4712^\circ$$

12. Which of the following is a unit vector in the same direction as $a = -3i + 4j$.

A. $-\frac{3}{5}i + \frac{4}{5}j$ B. $-3i + 4j$ C. $\frac{3}{5}i - \frac{4}{5}j$ D. $3i - 4j$

E. None of the above

$$u = \frac{1}{\|a\|} a$$

$$\|a\| = \sqrt{3^2 + 4^2}$$

$$\|a\| = \sqrt{25}$$

$$\|a\| = 5$$

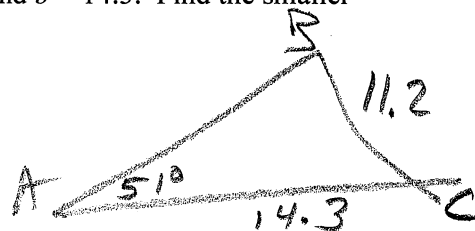
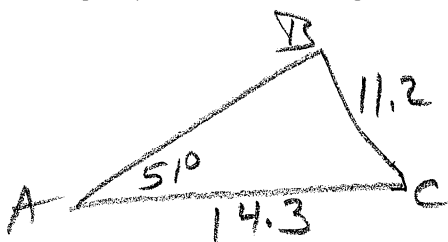
$$u = \frac{1}{5}(-3i + 4j)$$

$$= -\frac{3}{5}i + \frac{4}{5}j$$

13. There are two distinct $\triangle ABC$ with $\alpha = 51^\circ$, $a = 11.2$ and $b = 14.3$. Find the smaller value of two angles γ to the nearest degree.

A. $\gamma = 23^\circ$ B. $\gamma = 46^\circ$ C. $\gamma = 32^\circ$ D. $\gamma = 67^\circ$

E. None of the above



$$\frac{\sin 51^\circ}{11.2} = \frac{\sin B}{14.3}$$

$$\sin B = 0.9922$$

$$B = 51^\circ$$

$$B_1 = 82.86^\circ$$

$$\gamma_1 = 46.14^\circ$$

$$B_2 = 180^\circ - 82.86^\circ = 97.14^\circ$$

$$\gamma_2 = 31.86^\circ$$

Covering Section 7.4 starting at #35 and Sections 7.6, 8.1, 8.2, 8.3, and 8.4

14. Find the solutions of the equation that are in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ to four decimal places.

$$\tan^2 x - 3 \tan x - 5 = 0$$

A. 1.4467, -1.1365

B. 4.1926, -1.1926

C. 0.1876, -0.3817

D. 1.3367, -0.8730

E. None of the above

$$\tan x = \frac{3 \pm \sqrt{9 - 4(1)(-5)}}{2(1)} = \frac{3 \pm \sqrt{29}}{2}$$

$$\tan x = \frac{3 + \sqrt{29}}{2}$$

$$\tan x = \frac{3 - \sqrt{29}}{2}$$

$$\tan x = 4.1926$$

$$\tan x = -1.1926$$

$$x = \tan^{-1}(4.1926)$$

$$x = \tan^{-1}(-1.1926)$$

$$x = 1.3367$$

$$x = -0.8730$$

15. Write the expression as an algebraic expression in x for $x > 0$.

means QI

$$\sin\left(\arctan\left(\frac{x}{3}\right)\right)$$

 α

$$\rightarrow \sin \alpha$$

$$= \frac{x}{\sqrt{x^2 + 9}}$$

A. $\frac{x}{\sqrt{9+x^2}}$

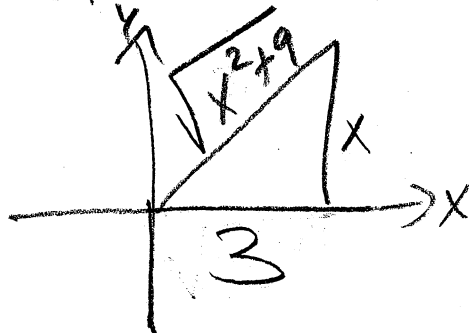
B. $\frac{3}{3+x}$

C. $\frac{3}{\sqrt{9+x^2}}$

D. $\frac{x}{3+x}$

E. None of the above

$$\alpha = \tan^{-1}\left(\frac{x}{3}\right)$$



$$c^2 = x^2 + 3^2$$

$$c = \sqrt{x^2 + 9}$$

Question	Exam 3 Answers
1.	B. $\langle -16, -3 \rangle$
2.	C. $0, \pi, \frac{3\pi}{4}, \frac{5\pi}{4}$
3.	D. 10.3 lb
4.	A. $\frac{-\pi}{3}$
5.	D. $\frac{-8\sqrt{33}}{49}$
6.	B. $m = \frac{14}{5}$
7.	C. 33.7
8.	D. 298 mph
9.	A. 197 miles
10.	C. 478 feet
11.	B. 122.5°
12.	A. $\frac{-3}{5}i + \frac{4}{5}j$
13.	C. $\gamma = 32^\circ$
14.	D. 1.3367, -0.8730
15.	A. $\frac{x}{\sqrt{9+x^2}}$