

1.
 - a) When is the dot product of two vectors equal to zero?
 - b) When is the cross product of two vectors equal to zero?

2. Give the vector equations.
 - a) A line through a point P .
 - b) A plane through a point P .
 - c) A sphere with center P .

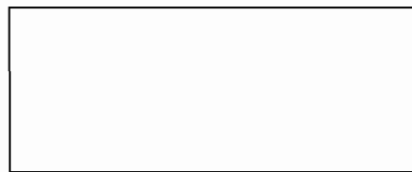
3. Here is the sketch of an ellipsoid. Write down its equation.

4. Write down an equation of the plane $x + y + z = 1$ in spherical coordinates.

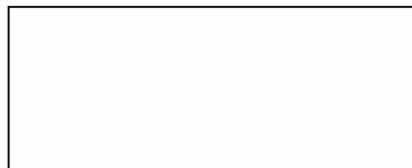
5. A rocket ship is at rest at the origin of space at time $t = 0$, and is accelerating at constant acceleration \vec{a} .
 - a) What is its position as a function of t ?
 - b) What kind of a curve is the rocket ship's trajectory?
 - c) What are \vec{T} , \vec{N} , \vec{B} and curvature κ of this trajectory?

6. A fly is traveling with velocity $\vec{v} = \vec{i} + \sqrt{2}t\vec{j} + t^2\vec{k}$
- What is the length of the fly's trajectory between $t = 0$ and $t = 1$?
 - What is the distance between the points the fly was at when $t = 0$ and $t = 1$?
 - What is the curvature of the fly's trajectory at $t = 0$?
7. The vector equation $\vec{r} = \frac{1}{1 + \cos \theta} \vec{u}_r$ describes an ellipse whose focus is at the origin. Express the tangent \vec{T} of the ellipse at the point where $\theta = \frac{\pi}{2}$ in terms of the orthonormal frame $\vec{u}_r, \vec{u}_\theta, \vec{k}$ (corresponding to cylindrical coordinates).
- Hint: $\frac{d\vec{u}_r}{d\theta} = \vec{u}_\theta$ and $\frac{d\vec{u}_\theta}{d\theta} = -\vec{u}_r$ and $\frac{d\vec{k}}{d\theta} = \vec{0}$.

1. Let D be the region bounded by the y -axis and the parabola $x = -4y^2 + 3$. Compute $\iint_D (2 + y) dA$.



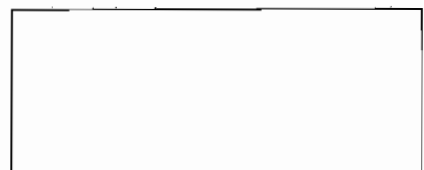
2. Determine $\int_C (2x^3 - y^3) dx + (x^3 + y^3) dy$, where C is the unit circle centered at the origin, oriented counterclockwise.



3. Evaluate $\int_0^9 \int_0^{\sqrt{81-y^2}} \int_0^{\sqrt{81-x^2-y^2}} \frac{dz dx dy}{x^2 + y^2 + z^2}.$



4. Find the volume of the region between the two elliptic paraboloids $z = \frac{1}{9}x^2 + y^2 - 4$ and $z = -\frac{1}{9}x^2 - y^2 + 4$.



5. Compute $\int_C (x + y)^5 ds$ along $C: x = t + 1/t, y = t - 1/t$, from $(2, 0)$ to $(17/4, 15/4)$.

