Math 174 Exam I 2/14/00 D. Gottlieb

1. a) When is the dot product of two vectors equal to zero?

   b) When is the cross product of two vectors equal to zero?

2. Give the vector equations.

   a) A line through a point P.

   b) A plane through a point P.

   c) A sphere with center P.

3. Here is the sketch of an ellipsoid. Write down its equation.

4. Write down an equation of the plane $x + y + z = 1$ in spherical coordinates.

5. A rocket ship is at rest at the origin of space at time $t = 0$, and is accelerating at constant acceleration $\vec{a}$.

   a) What is its position as a function of t?

   b) What kind of a curve is the rocket ship's trajectory?

   c) What are $\vec{T}$, $\vec{N}$, $\vec{B}$ and curvature κ of this trajectory?

6. A fly is traveling with velocity $\vec{v} = \vec{\imath} + \sqrt{2}t\vec{\jmath} + t^2\vec{k}$

   a) What is the length of the fly's trajectory between $t = 0$ and $t = 1$?

   b) What is the distance between the points the fly was at when $t = 0$ and $t = 1$?

   c) What is the curvature of the fly's trajectory at $t = 0$?

7. The vector equation $\vec{r} = \dfrac{1}{1+\cos\theta}\,\vec{u}_r$ describes an ellipse whose focus is at the origin. Express the tangent $\vec{T}$ of the ellipse at the point where $\theta = \dfrac{\pi}{2}$ in terms of the orthonormal frame $\vec{u}_r$, $\vec{u}_\theta$, $\vec{k}$ (corresponding to cylindrical coordinates).
   Hint: $\dfrac{d\vec{u}_r}{d\theta} = \vec{u}_\theta$ and $\dfrac{d\vec{u}_\theta}{d\theta} = -\vec{u}_r$ and $\dfrac{d\vec{k}}{d\theta} = \vec{0}$.

Math 174, Exam II
Spring 2000

Professor Donnelly

1. Let D be the region bounded by the y–axis and the parabola $x = -4y^2 + 3$. Compute $\displaystyle\iint_D (2+y)\,dA$.

2. Determine $\displaystyle\int_C (2x^3 - y^3)dx + (x^3 + y^3)dy$, where C is the unit circle centered at the origin, oriented counterclockwise.

3. Evaluate $\displaystyle\int_0^9 \int_0^{\sqrt{81-y^2}} \int_0^{\sqrt{81-x^2-y^2}} \frac{dz\,dx\,dy}{x^2+y^2+z^2}$.

4. Find the volume of the region between the two elliptic paraboloids $z = \frac{1}{9}x^2 + y^2 - 4$ and $z = -\frac{1}{9}x^2 - y^2 + 4$.

5. Compute $\int_C (x+y)^5 ds$ along C: $x = t + 1/t,\ y = t - 1/t$, from $(2,0)$ to $(17/4, 15/4)$.