Math 174, Final Exam
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Name ______________________

Show all your work. Use backs of pages if necessary.

(10 pts) 1. Let $f(x,y) = \ln(x^2+y^2) + \sqrt{1-x^2}$.

a. Find the domain of f.

b. Is the domain of f open, closed, or neither?

(10 pts) 2. The equation $xe^y - xz\cos y + 2z^2 = 2$ defines z implicitly as a function of x and y. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the point $(1,0,1)$.

(10 pts) 3. If $z = f(x, y)$, $x = u^2 + v^2$, and $y = u + v$, find $\dfrac{\partial z}{\partial u}$ at the point $(u, v) = (1, 2)$ given:

$$f_x(5,3) = 1, \; f_x(1,2) = 2, \; f_x(2,1) = 3,$$
$$f_y(5,3) = 4, \; f_y(1,2) = 5, \; f_y(2,1) = 6.$$

(15 pts) 4. Consider the surface $x^2 - xyz + z^2 = 1$.

a. Find an equation for the tangent plane at the point $(1, 1, 1)$.

b. Find a parametric equation for the normal line at the point $(1, 1, 1)$.

(12 pts) 5. Let $f(x, y, z) = x^2y - yz$ and $\mathbf{u} = \frac{2}{3}\,\mathbf{i} + \frac{1}{3}\,\mathbf{j} - \frac{2}{3}\,\mathbf{k}$.

a. Find the directional derivative of f at the point $(1, 1, -1)$ in the direction of $\mathbf{u}$.

b. Find a unit vector that gives the direction of most rapid decrease of f at the point $(1, 1, -1)$.

(15 pts) 6. Use Lagrange multipliers to find the maximum and minimum values of $x - 2y - 2z$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

(10 pts) 7. Provide the limits that reverse the order of integration in the following integral. DO NOT attempt to compute the integral. Write your answers in the boxes:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} f(x,y)\,dydx = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} f(x,y)\,dxdy.$$

(10 pts) 8. Set up an integral to determine the area of the plane region bounded by the curves $y^2 - x = 0$ and $y^3 - x = 0$. Write your answers in the boxes. DO NOT EVALUATE the integral.

$$A = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\, dxdy.$$

(10 pts) 9. Set up an integral to determine the mass of a body occupying the region in the first octant bounded by the coordinate planes and the plane $x + 2y + 3z = 6$. The density is $\delta(x, y, z) = xy^2z$. Write your answers in the boxes. DO NOT EVALUATE the integral.

$$M = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\, dzdydx.$$

(20 pts) 10. The region D is given by $x^2 + y^2 + z^2 \leq 1,\ z \geq \frac{1}{2}$.

a. Set up an integral in cylindrical coordinates to determine the volume of D. Write your answers in the boxes. DO NOT EVALUATE the integral.

$$V = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\, dzdrd\theta.$$