MATHEMATICS 182 TEST I

(35 pts) 1. Given the vectors $\vec{A} = i - 2j + k$ and $\vec{B} = i + 2j$ find:

a) A vector in the direction opposite from $\vec{A}$ of length 2,

b) The equation of a plane through $(1, 0, 2)$ perpendicular to $\vec{A}$,

c) The equations of a line through $(1, 0, 2)$ parallel to $\vec{B}$,

d) the area of the triangle generated by $\vec{A}$ and $\vec{B}$,

e) the equation of a plane through $(1, 0, 2)$ parallel to $\vec{A}$ and $\vec{B}$,

f) the intersection of the line of part c) with the plane

$$2x + 2y - z = 6.$$

2. If a curve is parametrized by

$$x = \frac{e^t \cos(t)}{\sqrt{2}}, \ y = \frac{e^t \sin(t)}{\sqrt{2}}, \ z = 6, \ -\infty < t < \infty,$$

find

a) The unit tangent vector, $\vec{T}$, as a function of t,

b) $\dfrac{ds}{dt}$ as a function of t,

c) $K = \left| \frac{d\vec{T}}{ds} \right|$ as a function of t.

d) $\vec{n}$ as a function of t.

e) $\vec{b}$ as a function of t.

(20 pts) 3) a) If z is defined as a function of x and y by the equation $xz + yz^3 - 2xy = 0$ find $\dfrac{\partial z}{\partial x}$ at $(1, 1, 1)$.

b) If $w = \sqrt{x^2 + y^2 + z^2}$, $x = u + v$, $y = u - v$, and $z = uv$ find $\dfrac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ when $u = 1$ and $v = 1$.

DO ONE OF PROBLEMS 4), 5), OR 6) (INDICATE YOUR CHOICE)

(15 pts) 4) Let $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$ where θ is the angle between $\vec{A}$ and $\vec{B}$.

If $\vec{A} = a_1 i + a_2 j + a_3 k$ and $\vec{B} = b_1 i + b_2 j + b_3 k$ show that $\vec{A}\circ\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$.

(15 pts) 5) Find the distance of the point $(0,3,1)$ from the plane defined by the two intersecting lines

$$x = 1 + 2t,\ y = 2 + t,\ \text{and } z = 1 - 2t,$$

and

$$x = 2 + s,\ y = 4 + 2s \quad z = 4 + 3s.$$

(15 pts) 6) If $\vec{T}, \vec{n}, \vec{b}$, are the tangent vector, the principal normal, and the binormal of a space curve, C,

a) Show that $\dfrac{d\vec{T}}{ds}\cdot\vec{T} = 0$.

b) Show that $\dfrac{d\vec{B}}{ds}\cdot\vec{B} = 0$.

c) Show that $\dfrac{d\vec{B}}{ds}$ is parallel to $\vec{n}$.