MATHEMATICS 182 TEST 2

(10 pts) 1) If $(x, y, z) = xe^y + z^2$ find

(a) the gradient, ∇f, at $(1, \ln 2, \frac{1}{2})$,

(b) the directional derivative of f in the direction of $\vec{V} = \left(\frac{1}{\sqrt{2}}\right)i + \left(\frac{1}{\sqrt{2}}\right)k$ at $\left(1, \ln 2, \frac{1}{2}\right)$.

(10 pts) 2) If z is defined as a function of x and y by

$$xy + z^3x - 2yz = 0$$

find $\dfrac{\partial z}{\partial x}$ at $(1, 1, 1)$.

(10 pts) 3) Find the tangent plane and normal line to $z - x^2 - y^2 = 1$ at $(2, 2, 5)$.

(10 pts) 4) If $f(x, y) = y(\sin x)$ find

(a) f_{xx}, f_{xy}, f_{yy} at $(0, 0)$

(b) the quadratic approximation of $f(x, y)$ at $(0, 0)$.

(15 pts) 5) If $z = \sin(xy) + x(\sin y)$,

$$x = u^2 + v^2, \text{ and } y = uv$$

find $\dfrac{\partial z}{\partial u}$ when $u = 0$ and $v = 1$.

(15 pts) 6) a) If $f(x, y) = xy^2 + y(\cos x)$ find the linearization $\ell(x, y)$ of $f(x, y)$ at $(0, 1)$.

b) Estimate the error if $|x| < \dfrac{1}{10}$ and $|y - 1| < \dfrac{1}{10}$.

(15 pts) 7) Find the absolute maximum value and the absolute minimum value of the function $f(x, y) = x^2 + xy + y^2$ on the rectangular plots $0 \leq x \leq 5$ and $-1 \leq y \leq 1$.

(15 pts) 8) Find the points on $x + 2y + 3z = 13$ closest to $(1, 1, 1)$.